Midterm Exam

Instructions: Please ask me if you do not understand something. I want you to understand.

You may not use the help of any notes or books or friends. Please!

Good luck, my students.

— Phil Rogaway

Your Name in English:

Your Name in Thai:

Your Nickname in Thai:

Your Student Number:

On problem	you got	out of
1		50
2		20
3		15
4		15
Σ		100

1 Short Answer [50 points]

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1. Draw a 3-state **DFA** for the language of all binary strings which encode numbers divisible by 3; that is, $L = 0^*\{\varepsilon, 11, 110, 1001, \cdots\}$.

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2. Draw a 4-state **DFA** for the language:

$L = \{x \in \{a, b\}^*$: the number of times ab appears in x is **even**$\}$.

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3. Write a regular expression for the language:

$L = \{x \in \{a, b\}^* : |x| \text{ is even and its first character is } a\}$.

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4. Write the following language as simply as you can:

$(a \cup abb^*a \cup bba \cup bb^*)^* \quad =$

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5. List the first **five** strings, in lexicographic order, in the **complement** of $(a \cup ab)^*$.

(For taking the complement, the alphabet is $\Sigma = \{a, b\}$. Remember that the lexicographic order of $\{a, b\}^* = \{\varepsilon, a, b, aa, ab, ba, bb, aaa, aab, \ldots\}$).

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6. Using the construction seen in class, convert the following **NFA** into a **DFA** for the same language:

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7. Using the construction seen in class, convert the following **NFA** into a **regular expression** for the same language:

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8. You are given DFAs $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$. Suppose you want to construct a DFA $M = (Q, \Sigma, \delta, q_0, F)$ for the language $L(M_1) - L(M_2)$.

(By $L(M_1) - L(M_2)$ I mean set difference — the set of all strings in $L(M_1)$ which are **not** in $L(M_2)$.)

Suppose you use the **product construction**, so $Q = Q_1 \times Q_2$. Then the **final state set**, F, for the machine M will be:

$$F = \{(p, q) \in Q_1 \times Q_2 : \boxed{} \}.$$

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9. Given a DFA $M = (Q, \Sigma, \delta, q_0, F)$, describe a **DFA** or **NFA**, $M' = (Q', \Sigma, \delta', q_0', F')$, such that $L(M') = L(M) \cup \{\epsilon\}$. (Please say if you are trying to give a $\boxed{\text{DFA}}$ or an $\boxed{\text{NFA}}$.)

2 Justified True or False [20 points]

Put an **X** through the **correct** box. When it says "Explain" provide a **brief** (but convincing) justification. Where appropriate, **make this justification a counterexample.**

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1. For every n, the language $L_n = \{0^i 1^i : \ i \leq n\}$ is regular.

True **False**

Explain:

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2. If L^* is regular, then L is regular, too. **True** **False**

Explain:

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3. $L = \{a^i b^{i+j} c^j : \ i, j \geq 0\}$ is regular. **True** **False**

Explain:

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4. Every regular language can be accepted by **some NFA** which has exactly 15 final states.

Explain: **True** **False**

3 A Lower Bound on DFA Size [15 points]

Let $L = \{a, aa, aaa\}$. Prove that L can not be recognized by a 4-state DFA. The alphabet is $\Sigma = \{a\}$.

4 A Closure Property [15 points]

Let L be a language over Σ. Define

$$\mathit{Enlarge}(L) = \{x \in \Sigma^* : \text{for some } y \in \Sigma^*, \quad xy \in L\}$$

Prove that if L is regular, then so is $\mathit{Enlarge}(L)$.

To do this, let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA for M. Construct a DFA $M' = (Q', \Sigma, \delta', q_0', F')$ for $\mathit{Enlarge}(L)$. Show the machine M' that you get if M is the following machine: