Problem Set 3 Solutions

Problem 1. *Page 86, Exercise 1.14, part (b).*

Problem 2. *Page 86, Exercise 1.16, part (b).*

I leave you to your own devices on these!

Problem 3. *Use the pumping lemma to show that the following language is not regular:*

$$L = \{0^m 1^n 0^{m+n} : m, n \geq 0\}$$

Suppose for contradiction that L were regular. Let N be the pumping length, as guaranteed by the pumping lemma. Select $s = 0^N 1^N 0^{2N} \in L$. We have that that $s = xyz$, for some x, y, and z, $y \in 0^+$, and $xy^i z \in L$ for all $i \geq 0$. But, letting $i = 0$, observe that $xz = 0^n 1^N 0^{2N}$, where $n < N$. This string is not in L, a contradiction.

Pumping up would work fine, too.

Problem 4. *Use the pumping lemma to show that the following language is not regular.*

$$L = \{x = y + z : x, y, \text{ and } z \text{ are binary numbers and } x \text{ is the sum of } y \text{ and } z\}$$

Suppose for contradiction that L were regular. Let N be the pumping length, as guaranteed by the pumping lemma. Consider the string s which is $1^N = 1^N + 0$. By the pumping lemma, s can be partitioned into $s = xyz$ where $|xy| \leq N$, $|y| \geq 1$, and $xy^i z \in L_c$ for all $i \geq 0$. But y then falls within the initial run of 1's and so $xy^0 z$ is a string $1^n = 1^N + 0$ for some $n < N$. This string is not in L, a contradiction.

Pumping up would work fine, too.

Problem 5. *Are the following propositions true or false? Support your answers with either proofs or counterexamples.*

(1) *If $L_1 \cup L_2$ is regular and L_1 is finite, then L_2 is regular.*

TRUE. Because $L_2 = (L_1 \cup L_2) - (L_1 - L_2)$. The first part is regular by assumption, the second part is regular because it is finite (being a finite set minus something), and regular languages are closed under set difference.

(Why are regular languages closed under set difference? Either because (a) you can see this using the product construction, or (b) you can rewrite set difference, $A - B$, as, $A \cap \overline{B}$, and regular languages are closed under intersection and complement.)

(2) *If $L_1 \cup L_2$ is regular and L_1 is regular, then L_2 is regular.*

FALSE. Let $L_1 = \Sigma^*$ and let L_2 be any nonregular language over Σ.

(3) *If $L_1 L_2$ is regular and L_1 is finite, then L_2 is regular.*

FALSE. Let $L_1 = \{\epsilon, 0\}$ and let $L_2 = 0(00)^* \cup \{0^{2^n} : n \geq 0\}$, say.

(4) *If $L_1 L_2$ is regular and L_1 is regular, then L_2 is regular.*

FALSE. The counterexample of (c) is good for this, too.

(4) *If L^* is regular then L is regular.*

FALSE. Let $L = \{a^n b^n : n \geq 0\} \cup \{a, b\}$.