

## Problem Set 6 — Due 21 September 2000

### Problem 1.

**Part A.** Show that

$$L_A = \{\langle M, k \rangle : M \text{ is a TM which accepts at least one string of length } k\}$$

is not decidable.

**Part B.** Prove that

$$L_B = \{\langle M, k \rangle : M \text{ is a TM that loops on at least one string of length } k\}$$

is not decidable.

**Part C.** Let

$$L_C = \{\langle M, k \rangle : M \text{ is a TM which accepts some string of length } k, \\ \text{but } M \text{ loops on some (other) string of length } k\}.$$

Show that  $L_C$  is not r.e.. (Assume the underlying alphabet has at least two characters.)

**Part D.** Show that  $L_C$  is not co-r.e., either.

**Problem 2.** Classify the following languages as **decidable**, **r.e.** (but not co-r.e.), **co-r.e.** (but not r.e.), or **neither** r.e. nor co-r.e.. Prove all your answers, giving decision procedures, acceptance procedures, or reductions.

**A.**  $L = \{\langle M \rangle : M \text{ accepts some even-length string}\}$ .

**B.**  $L = \{\langle M \rangle : M \text{ accepts some palindrome}\}$ .

**C.**  $L = \{\langle M \rangle : M \text{ never prints a "0" (regardless of the input)}\}$ .

**D.**  $L = \{\langle \alpha \rangle : \alpha \text{ is shortest regular expression for } L(\alpha)\}$ .