Image gradients and edges

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Last time

- Image formation
- Linear filters and convolution useful for
  - Image smoothing, removing noise
    - Box filter
    - Gaussian filter
    - Impact of scale / width of smoothing filter
- Separable filters more efficient
- Median filter: a non-linear filter, edge-preserving

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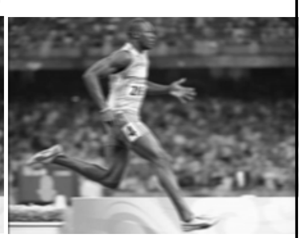
Review

Filter f = 1/9 x [1 1 1 1 1 1 1 1 1]

original image g



filtered

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Slide credit: Kristen Grauman

Review

Filter $f = 1/9 \times [1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1]^T$

original image g   filtered

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Slide credit: Kristen Grauman

Review

How do you sharpen an image?

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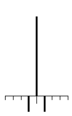
Practice with linear filters

Original

$$\begin{bmatrix}0&0&0\\0&2&0\\0&0&0\end{bmatrix} - \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$



Sharpening filter:
accentuates differences
with local average

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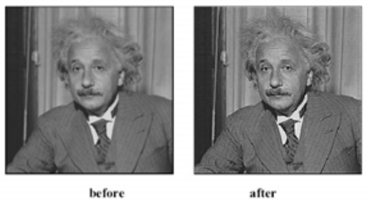
Slide credit: David Lowe

Filtering examples: sharpening



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Slide credit: Kristen Grauman

Sharpening revisited

What does blurring take away?

Let's add it back:

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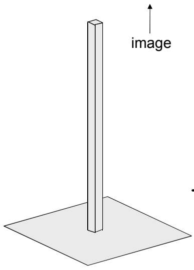
Slide credit: Svetlana Lazebnik

Unsharp mask filter

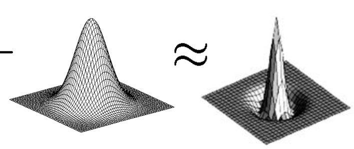
$$f + \alpha(f - f * g) = (1+\alpha) f - \alpha\, f * g = f * ((1+\alpha) e - g)$$

image; blurred image; unit impulse (identity)



unit impulse

—



Gaussian

≈

Laplacian of Gaussian

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Slide credit: Svetlana Lazebnik

Review

Median filter f:

Is f(a+b) = f(a)+f(b)?

Example:
a = [10 20 30 40 50]
b = [55 20 30 40 50]

Is f linear?

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Slide credit: Devi Parikh

Recall: Image filtering

- Compute a function of the local neighborhood at each pixel in the image
  - Function specified by a “filter” or mask saying how to combine values from neighbors
- Uses of filtering:
  - Enhance an image (denoise, resize, increase contrast, etc)
  - Extract information (texture, edges, interest points, etc)
  - Detect patterns (template matching)

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Slide credit: Kristen Grauman, Adapted from Derek Hoiem

Recall: Image filtering

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Slide credit: Kristen Grauman, Adapted from Derek Hoiem

Edge detection

- **Goal**: map image from 2d array of pixels to a set of curves or line segments or contours.
- **Why?**

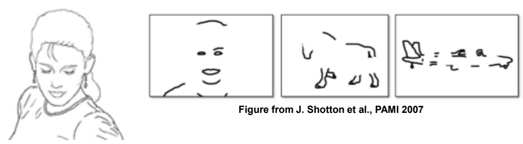

Figure from J. Shotton et al., PAMI 2007

- **Main idea**: look for strong gradients, post-process

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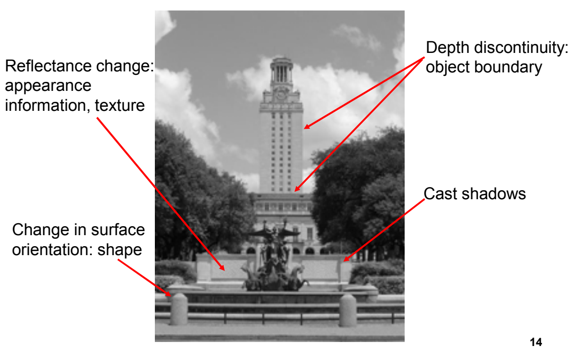
Slide credit: Kristen Grauman

What causes an edge?



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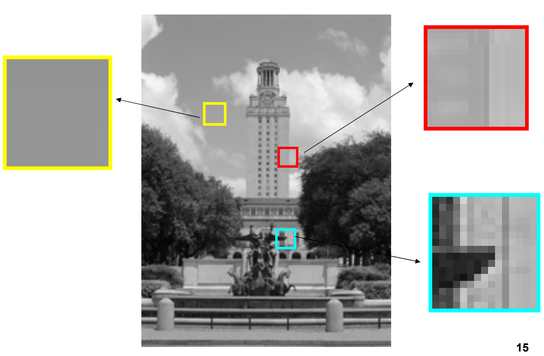
Edges/gradients and invariance



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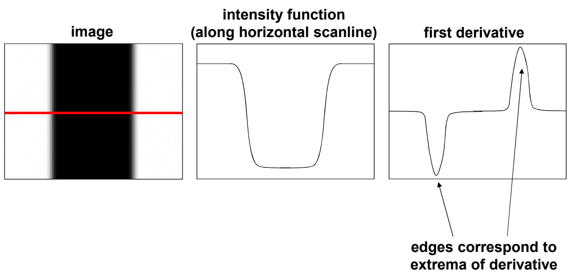
Derivatives and edges

An edge is a place of rapid change in the image intensity function.



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Slide credit: Svetlana Lazebnik

Derivatives with convolution

For 2D function, f(x,y), the partial derivative is:

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon, y) - f(x,y)}{\varepsilon}$$

For discrete data, we can approximate using finite differences:

$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+1, y) - f(x,y)}{1}$$

To implement above as convolution, what would be the associated filter?

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Partial derivatives of an image

$$\frac{\partial f(x,y)}{\partial x}$$

$$\frac{\partial f(x,y)}{\partial y}$$

-1	1

-1	? or	1
1		-1

Which shows changes with respect to x?

(showing filters for correlation)

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Slide credit: Kristen Grauman

Assorted finite difference filters

Prewitt: $M_x = \begin{bmatrix}-1 & 0 & 1\\ -1 & 0 & 1\\ -1 & 0 & 1\end{bmatrix}$; $M_y = \begin{bmatrix}1 & 1 & 1\\ 0 & 0 & 0\\ -1 & -1 & -1\end{bmatrix}$

Sobel: $M_x = \begin{bmatrix}-1 & 0 & 1\\ -2 & 0 & 2\\ -1 & 0 & 1\end{bmatrix}$; $M_y = \begin{bmatrix}1 & 2 & 1\\ 0 & 0 & 0\\ -1 & -2 & -1\end{bmatrix}$

Roberts: $M_x = \begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$; $M_y = \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$

```
>> My = fspecial('sobel');
>> outim = imfilter(double(im), My);
>> imagesc(outim);
>> colormap gray;
```

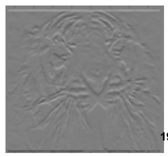

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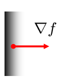
Image gradient

The gradient of an image:

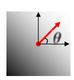
$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

The gradient points in the direction of most rapid change in intensity



$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$

$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$



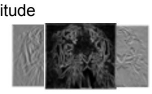
$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

The **gradient direction** (orientation of edge normal) is given by:

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

The **edge strength** is given by the gradient magnitude

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$



Slide credit: Steve Seitz

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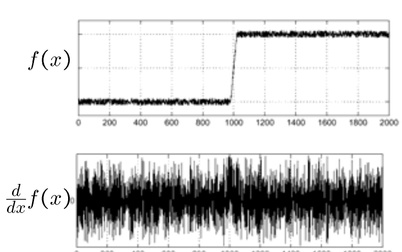
Effects of noise

Consider a single row or column of the image

- Plotting intensity as a function of position gives a signal



Where is the edge?

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Slide credit: Steve Seitz

Effects of noise

- Difference filters respond strongly to noise
  - Image noise results in pixels that look very different from their neighbors
  - Generally, the larger the noise the stronger the response
- What can we do about it?

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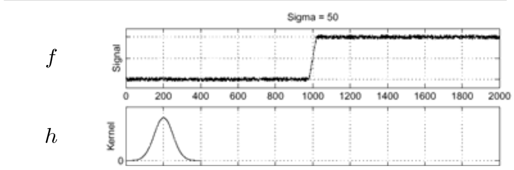
Source: D. Forsyth

Solution: smooth first



Where is the edge? Look for peaks in $\frac{\partial}{\partial x}(h \star f)$

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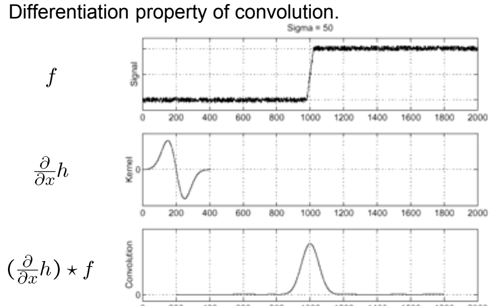
Derivative theorem of convolution

$$\frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x}h) \star f$$

Differentiation property of convolution.



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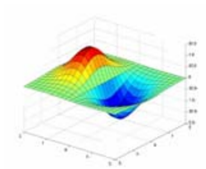
Slide credit: Steve Seitz

Derivative of Gaussian filters

$$(I \otimes g) \otimes h = I \otimes (g \otimes h)$$

0.0030	0.0133	0.0219	0.0133	0.0030
0.0133	0.0596	0.0983	0.0596	0.0133
0.0219	0.0983	0.1621	0.0983	0.0219
0.0133	0.0596	0.0983	0.0596	0.0133
0.0030	0.0133	0.0219	0.0133	0.0030

$$\otimes \begin{bmatrix} 1 & -1 \end{bmatrix}$$



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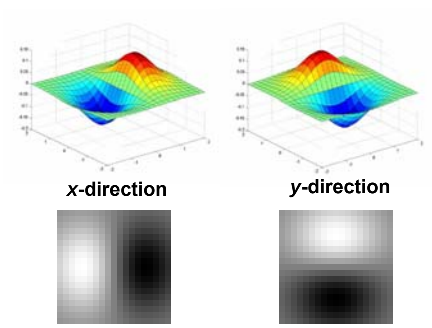
Slide credit: Kristen Grauman

Derivative of Gaussian filters



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Slide credit: Svetlana Lazebnik

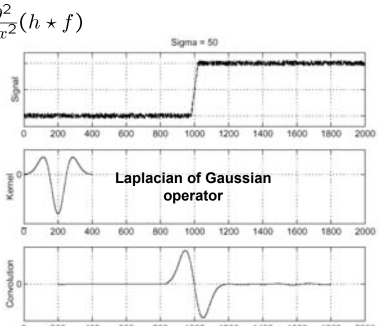
Laplacian of Gaussian

Consider $\frac{\partial^2}{\partial x^2}(h \star f)$

f

$\frac{\partial^2}{\partial x^2}h$

$(\frac{\partial^2}{\partial x^2}h) \star f$



Where is the edge? Zero-crossings of bottom graph

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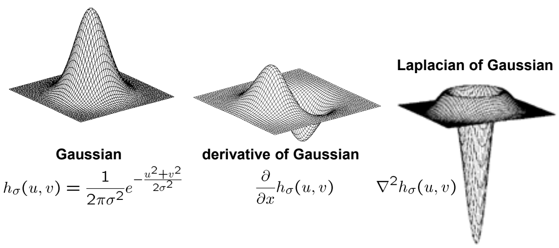
Slide credit: Steve Seitz

2D edge detection filters



$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$ $\quad \frac{\partial}{\partial x} h_\sigma(u,v)$ $\quad \nabla^2 h_\sigma(u,v)$

- ∇^2 is the Laplacian operator:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

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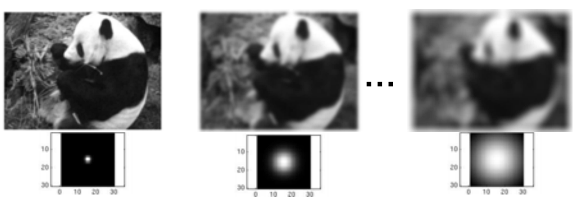
Slide credit: Steve Seitz

Smoothing with a Gaussian

Recall: parameter σ is the "scale" / "width" / "spread" of the Gaussian kernel, and controls the amount of smoothing.



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Effect of σ on derivatives

σ = 1 pixel σ = 3 pixels

The apparent structures differ depending on Gaussian's scale parameter.

Larger values: larger scale edges detected
Smaller values: finer features detected

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So, what scale to choose?

It depends what we're looking for.

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Mask properties

- <u>Smoothing</u>
  - Values positive
  - Sum to 1 → constant regions same as input
  - Amount of smoothing proportional to mask size
  - Remove "high-frequency" components; "low-pass" filter
- <u>Derivatives</u>
  - ___________ signs used to get high response in regions of high contrast
  - Sum to ___ → no response in constant regions
  - High absolute value at points of high contrast

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Slide credit: Kristen Grauman

Seam carving: main idea

[Shai & Avidan, SIGGRAPH 2007]

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Slide credit: Kristen Grauman

Seam carving: main idea

Content-aware resizing

Traditional resizing

[Shai & Avidan, SIGGRAPH 2007]

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Slide credit: Kristen Grauman

Seam carving: main idea

video

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Seam carving: main idea

Content-aware resizing

Intuition:

- Preserve the most “interesting” content
  - → Prefer to remove pixels with low gradient energy
- To reduce or increase size in one dimension, remove irregularly shaped “seams”
  - → Optimal solution via dynamic programming.

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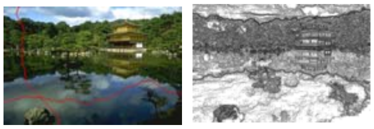
Slide credit: Kristen Grauman

Seam carving: main idea



$$Energy(f) = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

- Want to remove seams where they won't be very noticeable:
  - Measure "energy" as gradient magnitude
- Choose seam based on **minimum total energy path** across image, subject to 8-connectedness.

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Slide credit: Kristen Grauman

Seam carving: algorithm

$$Energy(f) = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Let a vertical seam **s** consist of *h* positions that form an 8-connected path.

Let the cost of a seam be: $Cost(\mathbf{s}) = \sum_{i=1}^{h} Energy(f(s_i))$

Optimal seam minimizes this cost: $\mathbf{s}^* = \min_{\mathbf{s}} Cost(\mathbf{s})$

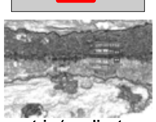
Compute it efficiently with dynamic programming. 38

Slide credit: Kristen Grauman

How to identify the minimum cost seam?

- How many possible seams are there?
  - height *h*, width *w*
- First, consider a **greedy** approach:

1	3	0
2	8	9
5	2	6



Energy matrix (gradient magnitude)

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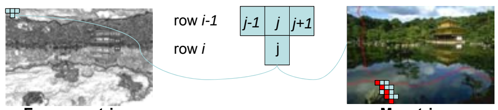
Slide credit: Adapted from Kristen Grauman

Seam carving: algorithm

- Compute the cumulative minimum energy for all possible connected seams at each entry *(i,j)*:

$$\mathbf{M}(i, j) = Energy(i, j) + \min(\mathbf{M}(i-1, j-1), \mathbf{M}(i-1, j), \mathbf{M}(i-1, j+1))$$



Energy matrix (gradient magnitude)

M matrix: cumulative min energy (for vertical seams)

- Then, min value in last row of **M** indicates end of the minimal connected vertical seam.
- Backtrack up from there, selecting min of 3 above in **M**.

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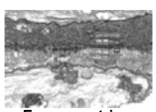
Slide credit: Kristen Grauman

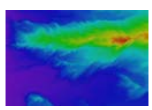
Example

$$\mathbf{M}(i, j) = Energy(i, j) + \min(\mathbf{M}(i-1, j-1), \mathbf{M}(i-1, j), \mathbf{M}(i-1, j+1))$$

1	3	0
2	8	9
5	2	6



Energy matrix (gradient magnitude)



M matrix (for vertical seams)

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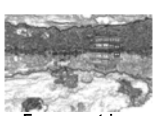
Slide credit: Kristen Grauman

Example

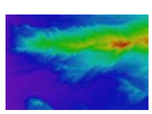
$$\mathbf{M}(i, j) = Energy(i, j) + \min(\mathbf{M}(i-1, j-1), \mathbf{M}(i-1, j), \mathbf{M}(i-1, j+1))$$

1	3	0
2	8	9
5	2	6

1	3	0
3	8	9
8	5	14



Energy matrix (gradient magnitude)



M matrix (for vertical seams)

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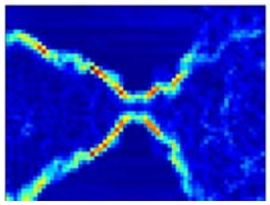
Slide credit: Kristen Grauman

Real image example

Original Image

Energy Map



Blue = low energy
Red = high energy

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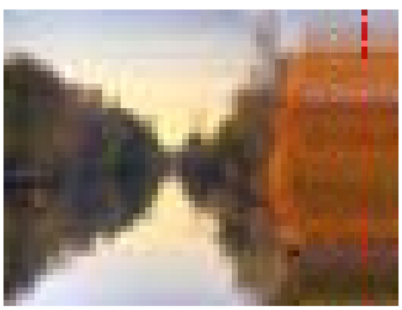
Slide credit: Kristen Grauman

Real image example



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Slide credit: Kristen Grauman

Other notes on seam carving

- Analogous procedure for horizontal seams
- Can also insert seams to *increase* size of image in either dimension
  - Duplicate optimal seam, averaged with neighbors
- Other energy functions may be plugged in
  - E.g., color-based, interactive,…
- Can use combination of vertical and horizontal seams

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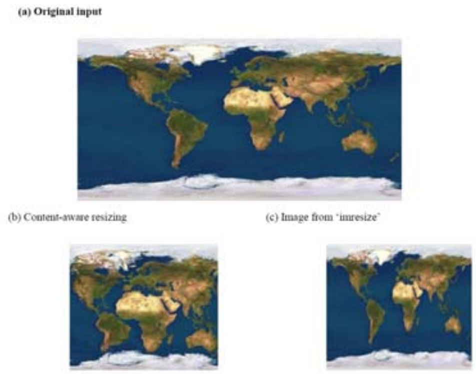
Slide credit: Kristen Grauman

Example seam carving results



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Original image

Conventional resize

Seam carving result

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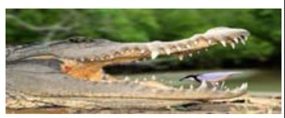
Original image



Conventional resize

Seam carving result

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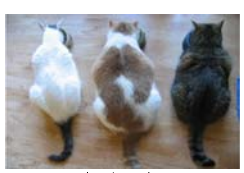
Original image (599 by 799)

Conventional resize (399 by 599)



Seam carving (399 by 599)

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Removal of a marked object

(a) Selected an area.

(b) Object is removed.

(c) Selected an area.

(d) Object is removed.

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Removal of a marked object

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Removal of a marked object

Figure A

Figure C

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“Failure cases” with seam carving

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“Failure cases” with seam carving

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Questions?

See you Tuesday!

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