

Linear Filters
April 7th, 2020

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UC Davis

Announcements

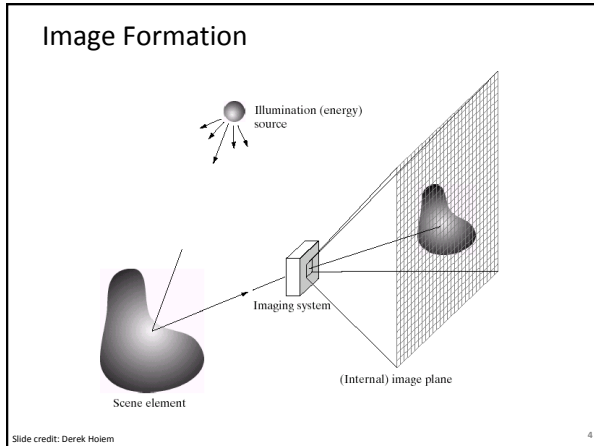
- PS0 due **4/10 Friday** at 11:59 pm
- Carefully read course website
- Sign-up for piazza

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Plan for today

- Image formation
- Image noise
- Linear filters
 - Examples: smoothing filters
- Convolution / correlation

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Digital camera

A digital camera replaces film with a sensor array

- Each cell in the array is light-sensitive diode that converts photons to electrons
- <http://electronics.howstuffworks.com/digital-camera.htm>

Slide credit: Steve Seitz

Digital images

- **Sample** the 2D space on a regular grid
- **Quantize** each sample (round to nearest integer)
- Image thus represented as a matrix of integer values.

The diagram shows a 2D image of a crumpled piece of paper being sampled on a regular grid. The resulting 2D matrix of integer values is shown below:

62	79	23	119	120	105	4	0
10	10	9	62	12	78	34	0
10	58	197	46	46	0	0	48
176	135	5	189	191	68	0	49
2	11	11	29	28	37	0	77
0	89	144	147	187	102	62	208
295	292	0	166	123	62	0	31
166	63	127	17	1	0	99	30

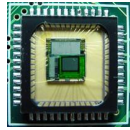
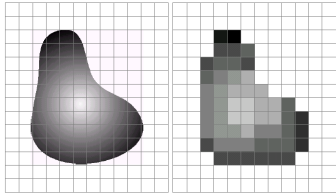
2D

The 2D matrix is then converted into a 1D signal, shown as a waveform and a bar chart labeled 'original'.

1D

Slide credit: Kristen Grauman, Adapted from Steve Seitz

Digital images



CMOS sensor

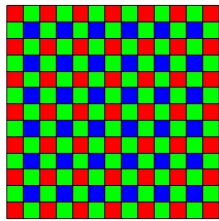
FIGURE 2.17 (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.

Semiconductor that records light electronically
Each sensor cell records amount of light coming in

Slide credit: Derek Hoiem

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Digital color images



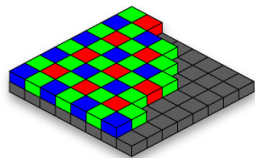
Bayer filter

© 2000 How Stuff Works

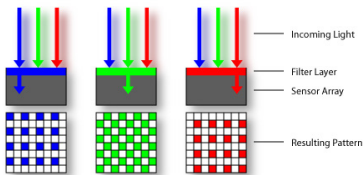
Slide credit: Kristen Grauman

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Color Sensing: Bayer Grid



Estimate RGB at each cell from neighboring values





http://en.wikipedia.org/wiki/Bayer_filter

Slide by Steve Seitz


Digital color images

Color images,
RGB color
space






R



G



B

Slide credit: Kristen Grauman 10

Images in Matlab

- Images represented as a matrix
- Suppose we have an NxM RGB image called "im"
 - `im(1,1,1)` = top-left pixel value in R-channel
 - `im(y, x, b)` = y pixels down, x pixels to right in the bth channel
 - `im(N, M, 3)` = bottom-right pixel in B-channel
- `imread(filename)` returns a uint8 image (values 0 to 255)
 - Convert to double format (values 0 to 1) with `im2double`

row	column	→ R	→ G	→ B						
0.92	0.93	0.94	0.97	0.62	0.37	0.85	0.97	0.93	0.92	0.99
0.95	0.89	0.82	0.89	0.56	0.31	0.75	0.92	0.81	0.95	0.91
0.89	0.72	0.51	0.35	0.51	0.42	0.57	0.41	0.49	0.91	0.92
0.96	0.56	0.98	0.94	0.56	0.46	0.91	0.87	0.90	0.97	0.95
0.71	0.81	0.81	0.87	0.57	0.37	0.80	0.88	0.89	0.79	0.85
0.49	0.62	0.60	0.58	0.50	0.60	0.58	0.50	0.61	0.45	0.33
0.86	0.84	0.74	0.58	0.51	0.39	0.71	0.92	0.91	0.49	0.74
0.96	0.67	0.54	0.85	0.48	0.37	0.88	0.90	0.94	0.82	0.93
0.69	0.49	0.56	0.66	0.43	0.42	0.77	0.73	0.71	0.90	0.99
0.79	0.73	0.90	0.67	0.33	0.61	0.69	0.79	0.73	0.93	0.97
0.91	0.94	0.85	0.49	0.41	0.78	0.76	0.77	0.89	0.99	0.93
0.79	0.73	0.90	0.67	0.33	0.61	0.69	0.79	0.73	0.93	0.97
0.91	0.94	0.89	0.49	0.41	0.78	0.76	0.77	0.89	0.99	0.93
0.79	0.73	0.90	0.67	0.33	0.61	0.69	0.79	0.73	0.93	0.97
0.91	0.94	0.89	0.49	0.41	0.78	0.76	0.77	0.89	0.99	0.93

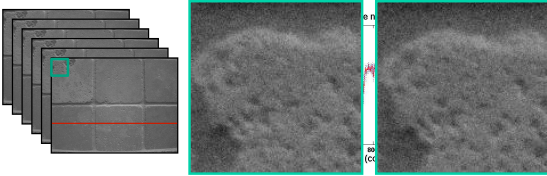
Slide credit: Derek Hoiem 11

Image filtering

- Compute a function of the local neighborhood at each pixel in the image
 - Function specified by a "filter" or mask saying how to combine values from neighbors
- Uses of filtering:
 - Enhance an image (denoise, resize, increase contrast, etc)
 - Extract information (texture, edges, interest points, etc)
 - Detect patterns (template matching)

Slide credit: Kristen Grauman, Adapted from Derek Hoiem 12

Motivation: noise reduction



- Even multiple images of the **same static scene** will not be identical.

Slide adapted from Kristen Grauman

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Common types of noise

- **Salt and pepper noise:** random occurrences of black and white pixels
- **Impulse noise:** random occurrences of white pixels
- **Gaussian noise:** variations in intensity drawn from a Gaussian normal distribution

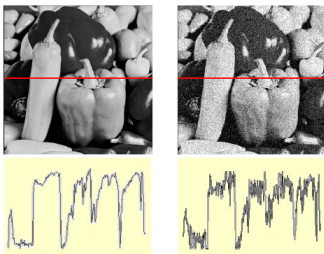


Original

Slide credit: Steve Seltz

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Gaussian noise



$$f(x, y) = \bar{f}(x, y) + \eta(x, y)$$
 Ideal Image Noise process Gaussian i.i.d. ("white") noise:
 $\eta(x, y) \sim \mathcal{N}(\mu, \sigma)$

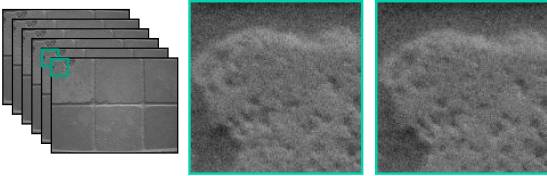
```
>> noise = randn(size(im)).*sigma;
>> output = im + noise;
```

What is impact of the sigma?

Figure from Martial Hebert

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Motivation: noise reduction



- Even multiple images of the same static scene will not be identical.
- How could we reduce the noise, i.e., give an estimate of the true intensities?
- **What if there's only one image?**

Slide credit: Kristen Grauman

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First attempt at a solution

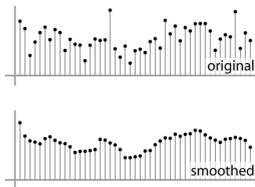
- Let's replace each pixel with an average of all the values in its neighborhood
- Assumptions:
 - Expect pixels to be like their neighbors
 - Expect noise processes to be independent from pixel to pixel

Slide credit: Kristen Grauman

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First attempt at a solution

- Let's replace each pixel with an average of all the values in its neighborhood
- Moving average in 1D:

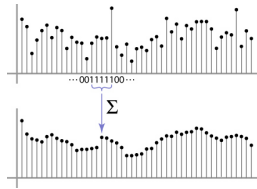


Slide credit: S. Marschoer

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Weighted Moving Average

Can add weights to our moving average
 Weights [1, 1, 1, 1, 1] / 5

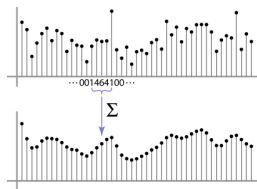


Slide credit: S. Marschner

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Weighted Moving Average

Non-uniform weights [1, 4, 6, 4, 1] / 16



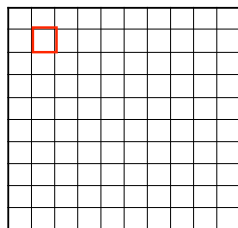
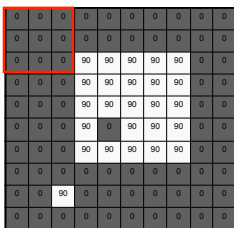
Slide credit: S. Marschner

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Moving Average In 2D

$f[x, y]$

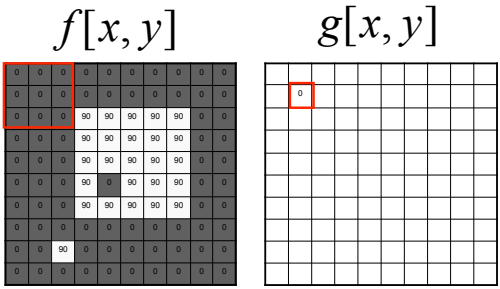
$g[x, y]$



Slide credit: Steve Seitz

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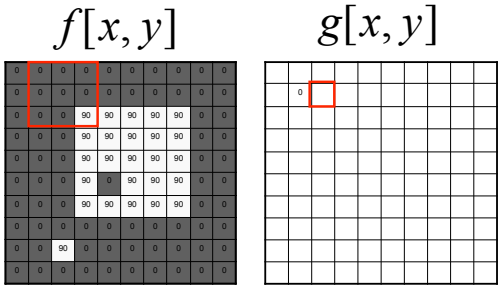
Moving Average In 2D



Slide credit: Steve Seltz

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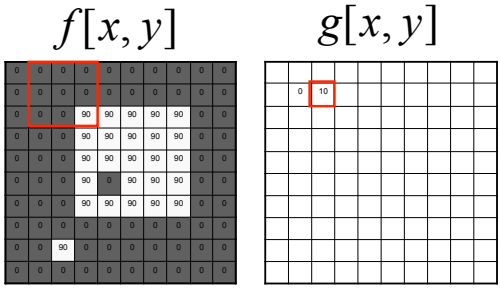
Moving Average In 2D



Slide credit: Steve Seltz

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Moving Average In 2D

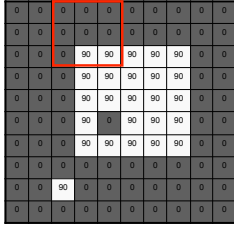


Slide credit: Steve Seltz

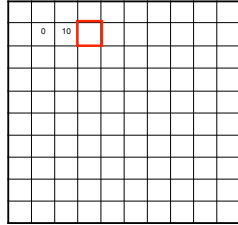
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Moving Average In 2D

$f[x, y]$



$g[x, y]$

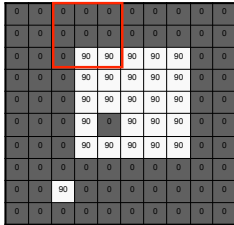


Slide credit: Steve Seltz

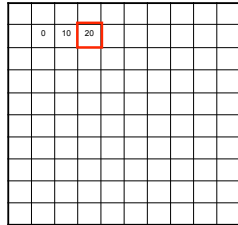
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Moving Average In 2D

$f[x, y]$



$g[x, y]$

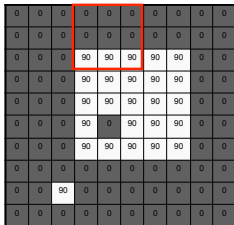


Slide credit: Steve Seltz

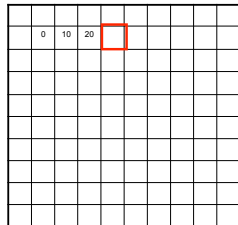
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Moving Average In 2D

$f[x, y]$



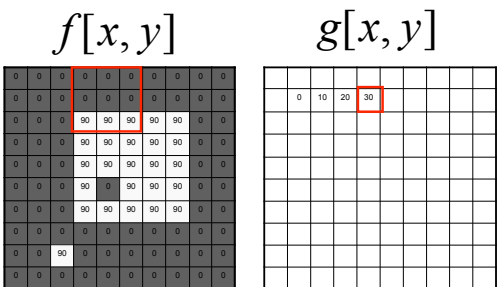
$g[x, y]$



Slide credit: Steve Seltz

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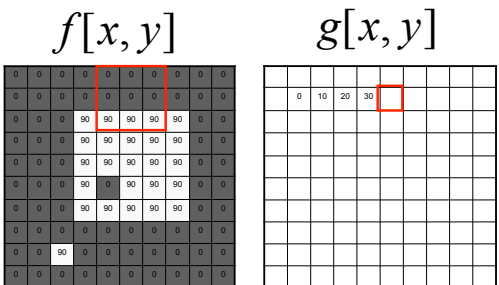
Moving Average In 2D



Slide credit: Steve Seltz

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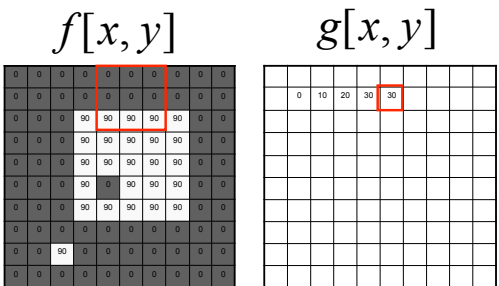
Moving Average In 2D



Slide credit: Steve Seltz

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Moving Average In 2D



Slide credit: Steve Seltz

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Moving Average In 2D

$f[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[x, y]$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	80	60	30	
	0	30	50	80	80	80	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	30	10	10	0	0	0	0	0	

Slide credit: Steve Seltz

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Correlation filtering

Say the averaging window size is $2k+1 \times 2k+1$:

$$g(i, j) = \frac{1}{(2k+1)^2} \sum_{u=-k}^k \sum_{v=-k}^k f(i+u, j+v)$$

Attribute uniform weight to each pixel Loop over all pixels in neighborhood around image pixel $f[i,j]$

Now generalize to allow **different weights** depending on neighboring pixel's relative position:

$$g(i, j) = \sum_{u=-k}^k \sum_{v=-k}^k \underbrace{h(u, v)}_{\text{Non-uniform weights}} f(i+u, j+v)$$

Slide adapted from Kristen Grauman

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Correlation filtering

$$g(i, j) = \sum_{u=-k}^k \sum_{v=-k}^k h(u, v) f(i+u, j+v)$$

This is called **cross-correlation**, denoted $g = h \otimes f$

Filtering an image: replace each pixel with a linear combination of its neighbors.

The filter "**kernel**" or "**mask**" $h[u,v]$ is the prescription for the weights in the linear combination.

Slide adapted from Kristen Grauman

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Averaging filter

- What values belong in the kernel h for the moving average example?

$f[x, y]$

\otimes
 $h[u, v]$
 $\frac{1}{9}$

“box filter”

$g[x, y]$

$g = h \otimes f$

Slide adapted from Kristen Grauman 34

Smoothing by averaging

depicts box filter:
white = high value, black = low value

original

filtered

What if the filter size was 5 x 5 instead of 3 x 3?

Slide credit: Kristen Grauman 35

Boundary issues

What about near the edge?

- the filter window falls off the edge of the image
- need to extrapolate
- methods:
 - clip filter (black)
 - wrap around
 - copy edge
 - reflect across edge

Slide credit: S. Marschoer 36

Boundary issues

What about near the edge?

- the filter window falls off the edge of the image
- need to extrapolate
- methods (MATLAB):
 - clip filter (black): `imfilter(f, g, 0)`
 - wrap around: `imfilter(f, g, 'circular')`
 - copy edge: `imfilter(f, g, 'replicate')`
 - reflect across edge: `imfilter(f, g, 'symmetric')`

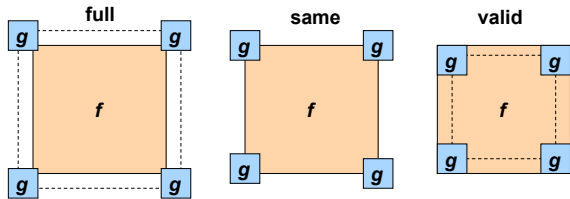
Slide credit: S. Marschner

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Boundary issues

What is the size of the output?

- MATLAB: output size / "shape" options
 - *shape* = 'full': output size is sum of sizes of f and g
 - *shape* = 'same': output size is same as f
 - *shape* = 'valid': output size is difference of sizes of f and g



Slide credit: Svetlana Lazebnik

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Gaussian filter

- What if we want nearest neighboring pixels to have the most influence on the output?

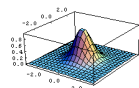
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$f[x, y]$$

$$\frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} h[u, v]$$

This kernel is an approximation of a 2d Gaussian function:

$$h(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$






- Removes high-frequency components from the image ("low-pass filter").

Slide credit: Steve Seitz

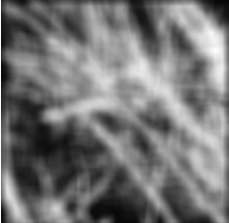


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Smoothing with a Gaussian



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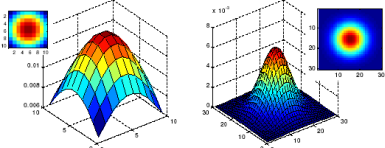
Smoothing with a box-filter



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Gaussian filters

- What parameters matter here?
- **Size** of kernel or mask
 - Note, Gaussian function has infinite support, but discrete filters use finite kernels



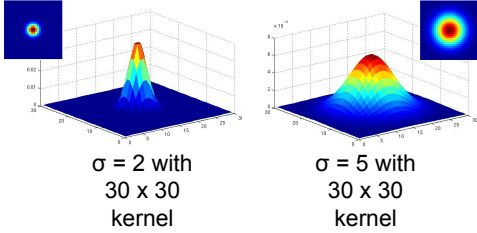
$\sigma = 5$ with 10 x 10 kernel $\sigma = 5$ with 30 x 30 kernel

Slide credit: Kristen Grauman

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Gaussian filters

- What parameters matter here?
- **Variance** of Gaussian: determines extent of smoothing





Slide credit: Kristen Grauman

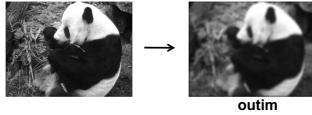
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Matlab

```
>> hsize = 30;
>> sigma = 5;
>> h = fspecial('gaussian', hsize, sigma);
```

```
>> mesh(h); 
>> imagesc(h); 
```

```
>> outim = imfilter(im, h); % correlation
>> imshow(outim);
```

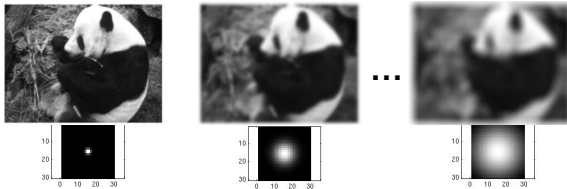


Slide credit: Kristen Grauman

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Smoothing with a Gaussian

Parameter σ is the “scale” / “width” / “spread” of the Gaussian kernel, and controls the amount of smoothing.



```
for sigma=1:3:10
  h = fspecial('gaussian', hsize, sigma);
  out = imfilter(im, h);
  imshow(out);
  pause;
end
```

Slide credit: Kristen Grauman

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Properties of smoothing filters

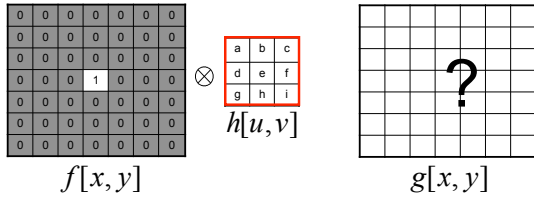
- Smoothing
 - Values positive
 - Sum to 1 → constant regions same as input
 - Amount of smoothing proportional to mask size
 - Remove "high-frequency" components; "low-pass" filter

Slide credit: Kristen Grauman

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Filtering an impulse signal

What is the result of filtering the impulse signal (image) f with the arbitrary kernel h ?



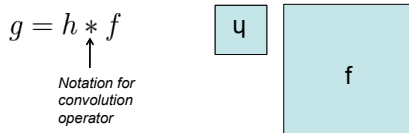
Slide adapted from Kristen Grauman

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Convolution

- Convolution:
 - Flip the filter in both dimensions (bottom to top, right to left)
 - Then apply cross-correlation

$$g(i, j) = \sum_{u=-k}^k \sum_{v=-k}^k h(u, v) f(i - u, j - v)$$



Slide adapted from Kristen Grauman

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Convolution vs. correlation

Convolution

$$g(i, j) = \sum_{u=-k}^k \sum_{v=-k}^k h(u, v) f(i - u, j - v)$$

$$g = h * f$$

Cross-correlation

$$g(i, j) = \sum_{u=-k}^k \sum_{v=-k}^k h(u, v) f(i + u, j + v)$$

$$g = h \otimes f$$

For a Gaussian or box filter, how will the outputs differ?
 If the input is an impulse signal, how will the outputs differ?

Slide adapted from Kristen Grauman

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Predict the outputs using correlation filtering

0	0	0
0	1	0
0	0	0

= ?

0	0	0
0	0	1
0	0	0

= ?

0	0	0
0	2	0
0	0	0

$\cdot \frac{1}{9}$

1	1	1
1	1	1
1	1	1

= ?

Slide credit: Kristen Grauman

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Practice with linear filters

0	0	0
0	1	0
0	0	0


?

Original


Slide credit: David Lowe

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Practice with linear filters



0	0	0
0	1	0
0	0	0

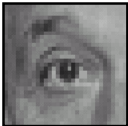


Original

Filtered
(no change)

Slide credit: David Lowe 52

Practice with linear filters



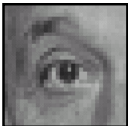
0	0	0
0	0	1
0	0	0

Original


?

Slide credit: David Lowe 53

Practice with linear filters



0	0	0
0	0	1
0	0	0

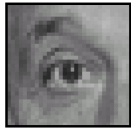


Original

Shifted left
by 1 pixel
with
correlation

Slide credit: David Lowe 54

Practice with linear filters



Original

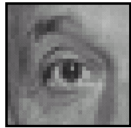
$$\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

?

Slide credit: David Lowe

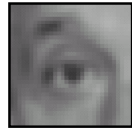
55

Practice with linear filters



Original

$$\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$



Blur (with a box filter)

Slide credit: David Lowe

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Practice with linear filters



Original

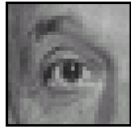
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

?

Slide credit: David Lowe

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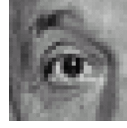
Practice with linear filters



0	0	0
0	2	0
0	0	0


 $-$
 $\frac{1}{9}$

1	1	1
1	1	1
1	1	1



Original

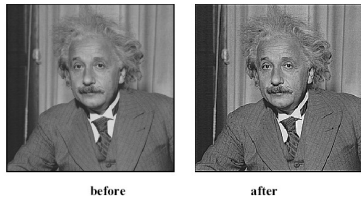
Sharpening filter:
accentuates differences
with local average



Slide credit: David Lowe

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Filtering examples: sharpening



Slide credit: Kristen Grauman

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Properties of convolution

- Shift invariant:
 - Operator behaves the same everywhere, i.e. the value of the output depends on the pattern in the image neighborhood, not the position of the neighborhood.
- Superposition:
 - $h * (f1 + f2) = (h * f1) + (h * f2)$

Slide credit: Kristen Grauman

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Properties of convolution

- Commutative:
 $f * g = g * f$
- Associative
 $(f * g) * h = f * (g * h)$
- Distributes over addition
 $f * (g + h) = (f * g) + (f * h)$
- Scalars factor out
 $kf * g = f * kg = k(f * g)$
- Identity:
 unit impulse $e = [\dots, 0, 0, 1, 0, 0, \dots]$. $f * e = f$

Slide credit: Kristen Grauman 61

Separability

- In some cases, filter is separable, and we can factor into two steps:
 - Convolve all rows
 - Convolve all columns

Slide credit: Kristen Grauman 62

Separability

- In some cases, filter is separable, and we can factor into two steps: e.g.,

g

1	2	1
---	---	---

h

2	3	3
3	5	5
4	4	6

What is the computational complexity advantage for a separable filter of size $k \times k$, in terms of number of operations per output pixel?


f

$f * (g * h) = (f * g) * h$

Slide credit: Kristen Grauman 63

Effect of smoothing filters

5x5



Additive Gaussian noise Salt and pepper noise

Slide credit: Kristen Grauman 64

Median filter

10	15	20
23	90	27
33	31	30

↓ Sort

10 15 20 23 27 30 31 33 90


↓ Replace

10	15	20
23	27	27
33	31	30

- No new pixel values introduced
- Removes spikes: good for impulse, salt & pepper noise
- Non-linear filter

Slide credit: Kristen Grauman 65

Median filter



Salt and pepper noise Median filtered

Matlab: `output im = medfilt2(im, [h w]);`

Slide credit: Martial Hebert 66


Median filter

- Median filter is edge preserving

	INPUT
	MEDIAN
	MEAN

Slide credit: Kristen Grauman 67

Filtering application: Hybrid Images



Slide credit: Kristen Grauman Aude Oliva & Antonio Torralba & Philippe G Schyns, SIGGRAPH 2006⁶⁸

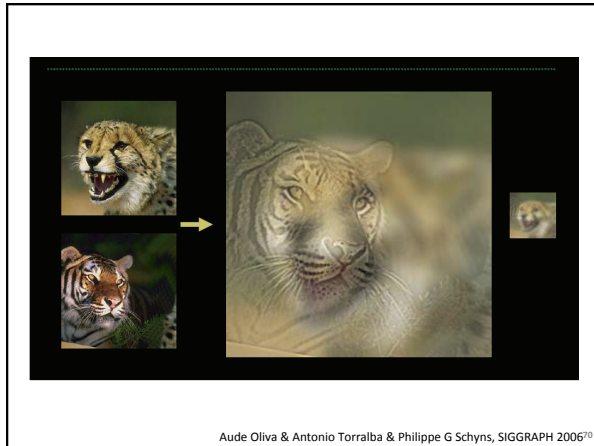
Application: Hybrid Images

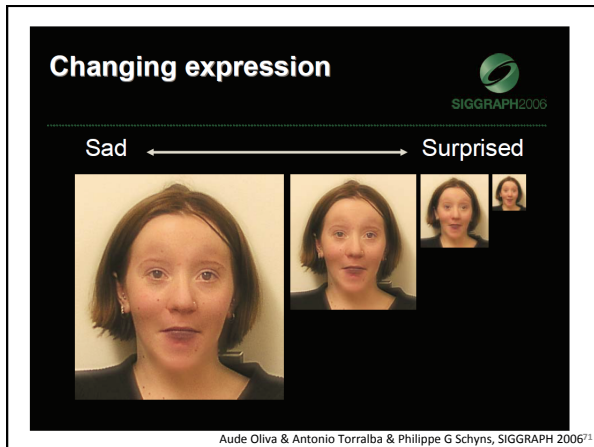
A. Oliva, A. Torralba, P.G. Schyns,
["Hybrid Images,"](#) SIGGRAPH 2006




unit

Slide credit: Kristen Grauman 69





Summary

- Image formation
- Image "noise"
- Linear filters and convolution useful for
 - Enhancing images (smoothing, removing noise)
 - Box filter
 - Gaussian filter
 - Impact of scale / width of smoothing filter
 - Detecting features (next time)
- Separable filters more efficient
- Median filter: a non-linear filter, edge-preserving

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Coming up

- Thursday:
 - Filtering part 2: filtering for features

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Questions?

See you Thursday!
