Fitting a transformation:
Feature-based alignment

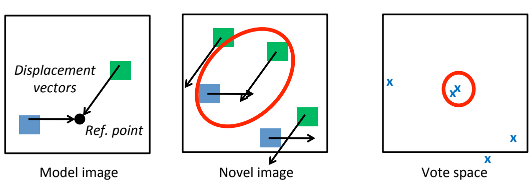
April 28th, 2020

Yong Jae Lee
UC Davis

Generalized Hough Transform

- What if we want to detect arbitrary shapes?

Intuition:



Now suppose those colors encode gradient directions…

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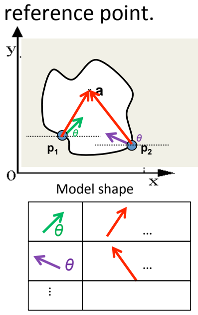
Generalized Hough Transform

- Define a model shape by its boundary points and a reference point.



Offline procedure:

At each boundary point, compute displacement vector: $\mathbf{r} = \mathbf{a} - \mathbf{p}_i$.

Store these vectors in a table indexed by gradient orientation θ.

[Dana H. Ballard, Generalizing the Hough Transform to Detect Arbitrary Shapes, 1980]

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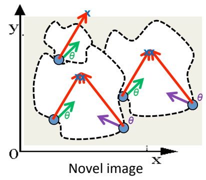
Generalized Hough Transform

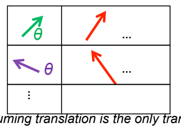
Detection procedure:

For each edge point:

- Use its gradient orientation ϑ to index into stored table
- Use retrieved **r** vectors to vote for reference point



Novel image



Assuming translation is the only transformation here, i.e., orientation and scale are fixed.

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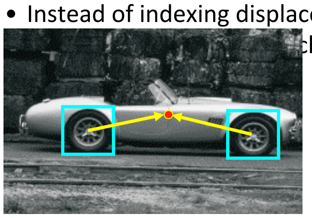
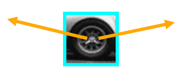
Generalized Hough for object detection

- Instead of indexing displacements by gradient ched local patterns.



training image



"visual codeword" with displacement vectors

B. Leibe, A. Leonardis, and B. Schiele,
Combined Object Categorization and Segmentation with an Implicit Shape Model,
ECCV Workshop on Statistical Learning in Computer Vision 2004

Source: L. Lazebnik

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Generalized Hough for object detection

- Instead of indexing displacements by gradient orientation, index by "visual codeword"

test image

B. Leibe, A. Leonardis, and B. Schiele,
Combined Object Categorization and Segmentation with an Implicit Shape Model,
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Source: L. Lazebnik

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Summary

- **Fitting** problems require finding any supporting evidence for a model, even within clutter and missing features
  - associate features with an explicit model
- **Voting** approaches, such as the **Hough transform**, make it possible to find likely model parameters without searching all combinations of features
  - Hough transform approach for lines, circles, …, arbitrary shapes defined by a set of boundary points, recognition from patches

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Today

- Feature-based alignment
  - 2D transformations
  - Affine fit
  - RANSAC

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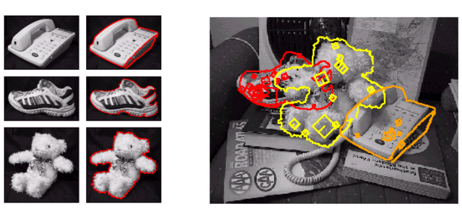
Motivation: Recognition



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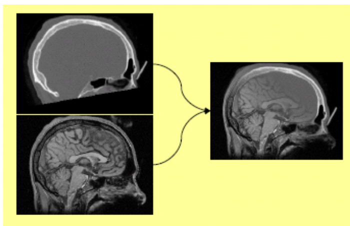
Figures from David Lowe

Motivation: medical image registration



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Slide credit: Kristen Grauman

Motivation: mosaics

(In detail next week)

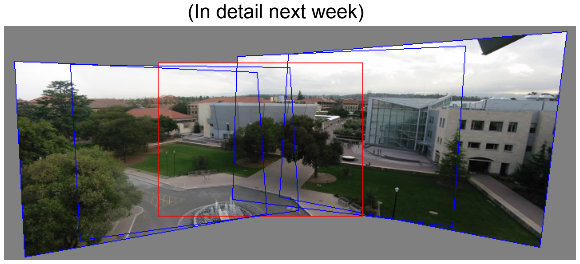

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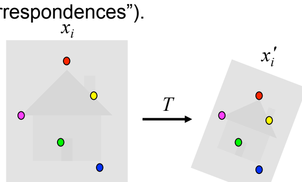
Image from http://graphics.cs.cmu.edu/courses/15-463/2010_fall/

Alignment problem

- We have previously considered how to **fit a model to image evidence**
  - e.g., a line to edge points
- In alignment, we will **fit the parameters of some transformation** according to a set of matching feature pairs ("correspondences").



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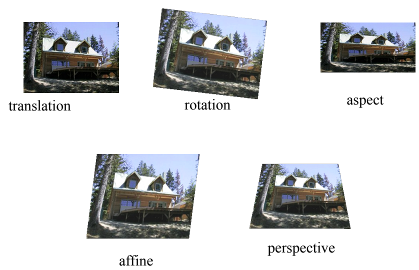
Parametric (global) warping

Examples of parametric warps:



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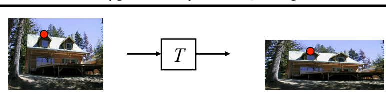
Source: Alyosha Efros

Parametric (global) warping



p = (x,y) **p'** = (x',y')

Transformation T is a coordinate-changing machine:

p' = *T*(p)

What does it mean that *T* is **global**?

- Is the same for any point p
- can be described by just a few numbers (parameters)

Let's represent *T* as a matrix:

p' = **M**p

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{M} \begin{bmatrix} x \\ y \end{bmatrix}$$

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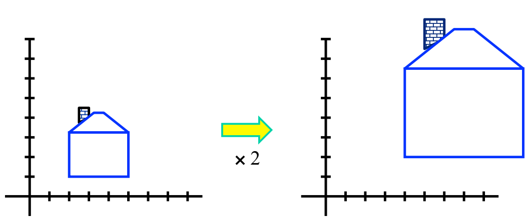
Source: Alyosha Efros

Scaling

Scaling a coordinate means multiplying each of its components by a scalar

Uniform scaling means this scalar is the same for all components:



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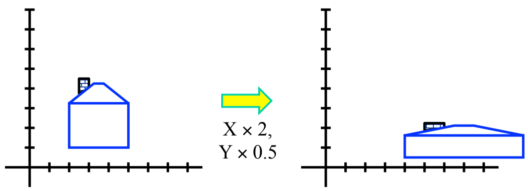
Source: Alyosha Efros

Scaling

Non-uniform scaling: different scalars per component:



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Source: Alyosha Efros

Scaling

Scaling operation:

$$x' = ax$$
$$y' = by$$

Or, in matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}}_{\textit{scaling matrix S}} \begin{bmatrix} x \\ y \end{bmatrix}$$

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Source: Alyosha Efros

What transformations can be represented with a 2x2 matrix?

2D Scaling?

$$x' = s_x * x \qquad y' = s_y * y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Rotate around (0,0)?

$$x' = \cos\Theta * x - \sin\Theta * y$$
$$y' = \sin\Theta * x + \cos\Theta * y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Shear?

$$x' = x + sh_x * y \qquad y' = sh_y * x + y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & sh_x \\ sh_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

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Source: Alyosha Efros

What transformations can be represented with a 2x2 matrix?

2D Mirror about Y axis?

$$x' = -x \qquad y' = y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Mirror over (0,0)?

$$x' = -x \qquad y' = -y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Translation?

$$\boldsymbol{x}' = \boldsymbol{x} + \boldsymbol{t}_x$$

$$\boldsymbol{y}' = \boldsymbol{y} + \boldsymbol{t}_y$$

NO!

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Source: Alyosha Efros

2D Linear Transformations

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Only linear 2D transformations can be represented with a 2x2 matrix.

Linear transformations are combinations of …

- Scale,
- Rotation,
- Shear, and
- Mirror

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Source: Alyosha Efros

Homogeneous coordinates

Convenient coordinate system to represent many useful transformations

To convert to homogeneous coordinates:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

Converting *from* homogeneous coordinates:

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

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Slide credit: Kristen Grauman

Homogeneous Coordinates

Q: How can we represent 2d translation as a 3x3 matrix using homogeneous coordinates?

$$x' = x + t_x$$
$$y' = y + t_y$$

A: Using the rightmost column:

$$\mathbf{Translation} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

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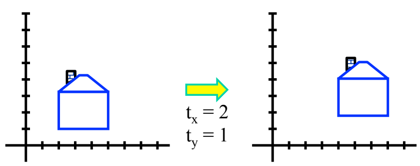
Source: Alyosha Efros

Translation

Homogeneous Coordinates

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$



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Basic 2D Transformations

Basic 2D transformations as 3x3 matrices

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Shear

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Source: Alyosha Efros

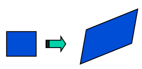
2D Affine Transformations

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Affine transformations are combinations of …

- Linear transformations, and
- Translations

Parallel lines remain parallel



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Today

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Alignment problem

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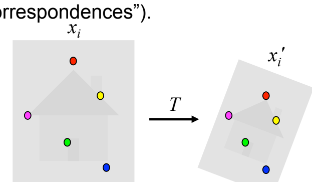

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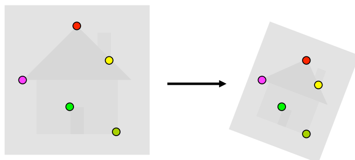
Image alignment



- Two broad approaches:
  - Direct (pixel-based) alignment
    - Search for alignment where most pixels agree
  - Feature-based alignment
    - Search for alignment where *extracted features* agree
    - Can be verified using pixel-based alignment

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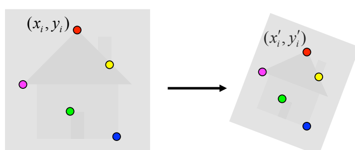
Fitting an affine transformation

- Assuming we know the correspondences, how do we get the transformation?



$$\begin{bmatrix} x_i' \\ y_i' \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$

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Slide credit: Kristen Grauman

An aside: Least Squares Example

Say we have a set of data points (X1,X1'), (X2,X2'), (X3,X3'), etc. (e.g. person's height vs. weight)

We want a nice compact formula (a line) to predict X's from Xs: Xa + b = X'

We want to find a and b

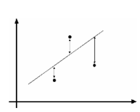
How many (X,X') pairs do we need?

$$X_1 a + b = X_1' \qquad \begin{bmatrix} X_1 & 1 \\ X_2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} X_1' \\ X_2' \end{bmatrix} \quad \text{Ax=B}$$

$$X_2 a + b = X_2'$$

What if the data is noisy?

$$\begin{bmatrix} X_1 & 1 \\ X_2 & 1 \\ X_3 & 1 \\ \dots & \dots \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} X_1' \\ X_2' \\ X_3' \\ \dots \end{bmatrix} \qquad \min \|Ax - B\|^2$$



overconstrained

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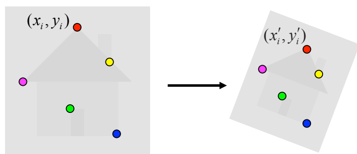
Source: Alyosha Efros

Fitting an affine transformation

- Assuming we know the correspondences, how do we get the transformation?



$$\begin{bmatrix} x_i' \\ y_i' \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$

$$\begin{bmatrix} \quad \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

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Slide credit: Kristen Grauman

Fitting an affine transformation

$$\begin{bmatrix} & & \cdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \cdots & & & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \cdots \\ x_i' \\ y_i' \\ \cdots \end{bmatrix}$$

- How many matches (correspondence pairs) do we need to solve for the transformation parameters?

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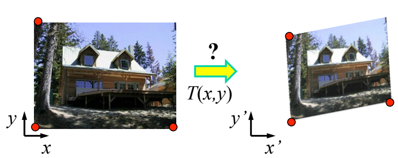
Affine: # correspondences?



How many correspondences needed for affine?

Alyosha Efros

Fitting an affine transformation

$$\begin{bmatrix} & & \cdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \cdots & & & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \cdots \\ x_i' \\ y_i' \\ \cdots \end{bmatrix}$$

- How many matches (correspondence pairs) do we need to solve for the transformation parameters?
- Once we have solved for the parameters, how do we compute the coordinates of the corresponding point for (x_{new}, y_{new}) ?
- Where do the matches come from?

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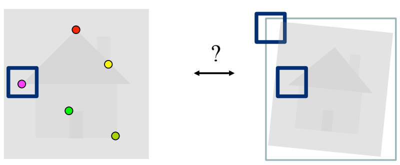
What **are** the correspondences?



- Compare content in **local** patches, find best matches.
  e.g., simplest approach: scan with template, and compute SSD or correlation between list of pixel intensities in the patch
- Later in the course: how to select regions using more robust descriptors.

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Fitting an affine transformation

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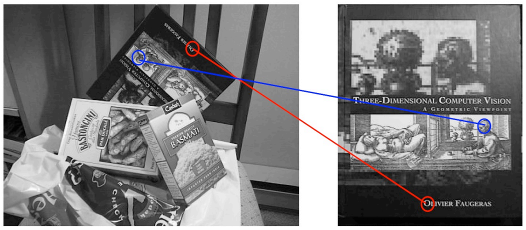
Figures from David Lowe, ICCV 1999

Fitting an affine transformation



Example from UBC SIFT Demo

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Fitting an affine transformation

Example from UBC SIFT Demo

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Fitting an affine transformation

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Figures from David Lowe, ICCV 1999

Today

- Feature-based alignment
  - 2D transformations
  - Affine fit
  - RANSAC

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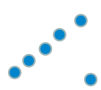
Outliers

- **Outliers** can hurt the quality of our parameter estimates, e.g.,
  - an erroneous pair of matching points from two images
  - an edge point that is noise, or doesn't belong to the line we are fitting.



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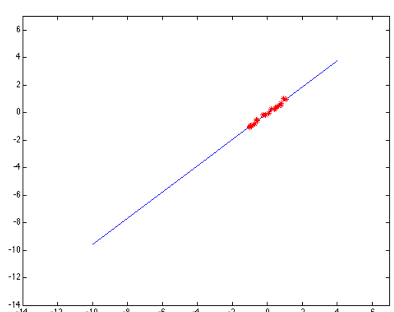
Outliers affect least squares fit



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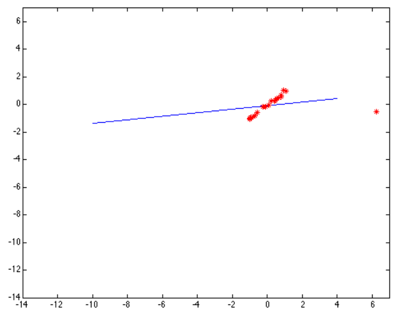
Slide credit: Kristen Grauman

Outliers affect least squares fit



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RANSAC

- RANdom Sample Consensus
- **Approach**: we want to avoid the impact of outliers, so let's look for "inliers", and use those only.
- **Intuition**: if an outlier is chosen to compute the current fit, then the resulting line won't have much support from rest of the points.

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RANSAC: General form

- RANSAC loop:

1. Randomly select a *seed group* of points on which to base transformation estimate
2. Compute transformation from seed group
3. Find *inliers* to this transformation
4. If the number of inliers is sufficiently large, re-compute estimate of transformation on all of the inliers

- Keep the transformation with the largest number of inliers

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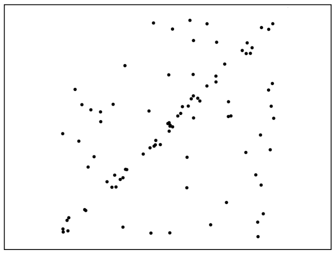
RANSAC for line fitting example



Source: R. Raguram

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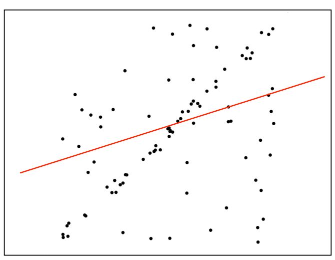
RANSAC for line fitting example



Least-squares fit

Source: R. Raguram

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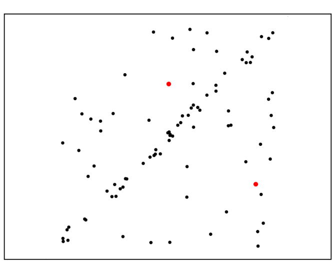
RANSAC for line fitting example



1. Randomly select minimal subset of points

Source: R. Raguram

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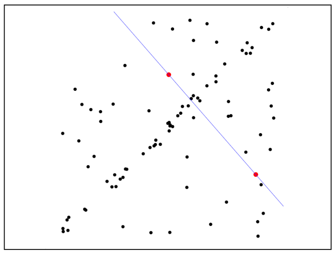
RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model

Source: R. Raguram

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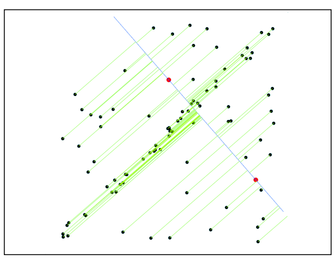
RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function

Source: R. Raguram

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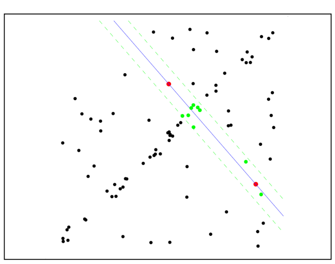
RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model

Source: R. Raguram

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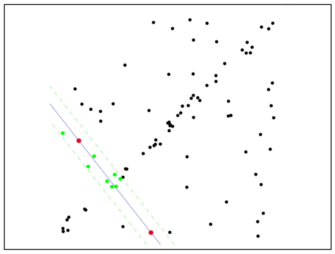
RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model
5. Repeat *hypothesize-and-verify* loop

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Source: R. Raguram

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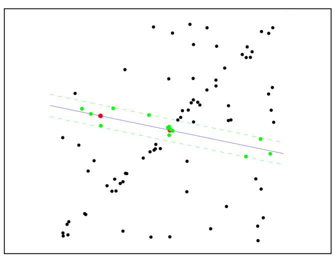
RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model
5. Repeat *hypothesize-and-verify* loop

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Source: R. Raguram

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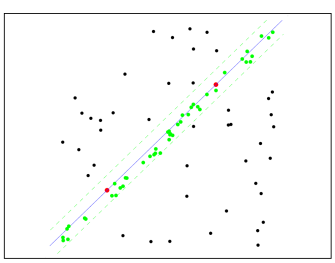

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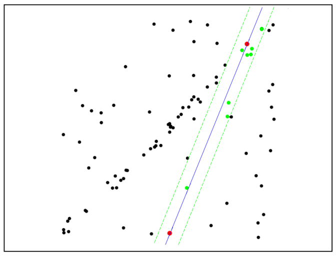

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Source: R. Raguram

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RANSAC for line fitting

Repeat ***N*** times:

- Draw ***s*** points uniformly at random
- Fit line to these ***s*** points
- Find inliers to this line among the remaining points (i.e., points whose distance from the line is less than ***t***)
- If there are ***d*** or more inliers, accept the line and refit using all inliers

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RANSAC pros and cons

- Pros
  - Simple and general
  - Applicable to many different problems
  - Often works well in practice
- Cons
  - Lots of parameters to tune
  - Doesn't work well for low inlier ratios (too many iterations, or can fail completely)

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Today

- Feature-based alignment
  - 2D transformations
  - Affine fit
  - RANSAC

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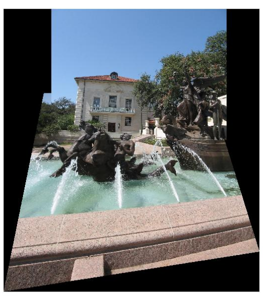
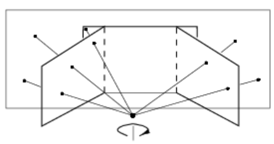
Coming up: alignment and image stitching





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Questions?

See you Thursday!

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