

# Fuzzy case-based reasoning for facial expression recognition

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## Abstract

Fuzzy logic (FL) and case-based reasoning (CBR) are two well-known techniques for the implementation of intelligent classification systems. Each technique has its own advantages and drawbacks. FL, for example, provides an intuitive user interface, simplifies the process of knowledge representation, and minimizes the system's computational complexity in terms of time and memory usage. On the other hand, FL has problems in knowledge elicitation which render it difficult to adopt for intelligent system implementation. CBR avoids these problems by making use of past input–output data to decide the system output for the present input. The accuracy of CBR system grows as the number of cases increase. However, more cases can mean added computational complexity in terms of space and time. In this paper we make the proposition that a hybrid system comprising a blend of FL and CBR can lead to a solution where the two approaches cover each other's weaknesses and benefit from each other's strengths. We support our claim by taking the problem of facial expression recognition from an input image. The facial expression recognition system presented in this paper uses a case base populated with fuzzy rules for recognizing each expression. Experimental results demonstrate that the system inherits the strengths of both methods.

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## 1. Introduction

The last few decades have seen a proliferation of intelligent systems for classification, diagnosis, and related tasks. Intelligent systems for classification have been used in a variety of domains: finance, biometrics, human computer interfaces, industrial management, medical diagnosis, software and hardware engineering. Despite the great variety of applications and approaches used for implementation of these systems, all of them address the same underlying issue of pattern classification: assigning a class or a label to a given set of observations. Traditionally these pattern classification tasks have been performed by statistical methods and models. Intelligent methods offer several advantages as compared to these traditional methods, including flexibility and an adaptation to the context of application.

Fuzzy logic (FL) and case-based reasoning (CBR) are two well-known techniques for the implementation of intelligent systems. These techniques share some common concepts: both involve selection, ranking, and aggregation of several alternatives for solving a particular problem. However, there are certain aspects in which the two approaches

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are distinct from each other, and have their own advantages and drawbacks. FL, for instance, simplifies the process of knowledge representation by employing the concept of a linguistic variable; a variable that can assume linguistic values like hot, near, and high. Linguistic variables, implemented with the help of fuzzy sets greatly reduce the system's knowledge base as an entire range of parameter values can be compactly represented by a single fuzzy set. FL uses these linguistic variables to define the system's knowledge base as a collection of fuzzy IF–THEN rules. The linguistic interface and simplified knowledge representation make FL an attractive choice for intelligent system implementation. However, one hurdle in the adoption of FL for intelligent system implementation is the difficulty of *knowledge elicitation*. FL-based systems obtain domain knowledge from domain experts to prepare the rules in the system's knowledge base. Nonetheless, there is no easy way to map the experts' knowledge to the system's rules. There are a lot of hedges that qualify the experts' decision making process which cannot be captured by the system's parameters.

CBR gets around this knowledge elicitation problem by keeping a historical repository of experience. Unlike the FL case, the database in a CBR-based intelligent system is composed of *cases*—comprising of (values of) input parameters encountered in the past and the corresponding system output. Any new input parameter configuration is decided upon by comparing with all the existing cases, and using the most similar case to guide the output decision. This decision and the corresponding input values are made part of the knowledge base for use in future decision making. Thus the knowledge base and accuracy of the system grows with experience. Conversely, a growth in knowledge base size also means a growth in the system's complexity in the context of computational time and memory requirements. As each encountered case is represented by its own set of crisp values for the parameters, an exponential growth in the size of the knowledge base is required to handle all possible cases.

In this paper we make the proposition that a hybrid system comprising a blend of FL and CBR can lead to a solution where the two approaches cover each other's weaknesses and benefit from each other's strengths. We believe that a hybrid FL/CBR system will be more accurate, easier to use and simpler in complexity as compared to the systems using any one of these approaches. We support our claim by taking the problem of facial expression recognition from an input image. The facial expression recognition system presented in this paper uses fuzzy linguistic variables to model various facial actions constituting prototypical facial expressions. The system uses a case base populated with fuzzy rules for recognizing each expression. Experimental results demonstrate that the system inherits the strengths of both methods, showing better performance than using either FL or CBR alone.

Facial expressions play a vital role in social communication. Research has shown that facial expressions help coordinate conversation and have a greater effect on whether a listener feels liked or disliked than the speaker's spoken words [5,44]. Computers are increasingly becoming the part of human social circle through advanced human–computer interfacing approaches. The human–computer interface is a bidirectional communication passage, i.e. the user and the machine should be able to understand each other using multiple modalities and respond in an intelligent manner. However, till the recent past this interaction has been one sided, i.e. from humans to computers. Communication is also required in the reverse direction to achieve maximum benefits from this interaction. Computers need to understand human emotions in order to respond and react correctly to human actions. Human face is the richest source of human emotions. So, facial expression recognition is the key to understanding human emotions. Recognition of facial expressions from facial gestures is very important for intelligent human–computer interaction.

Facial expression recognition presents an interesting research challenge for a variety of reasons. Human beings can exhibit a wide range of facial expressions. Methods are required to model this variety of expressions in a compact manner. The uniqueness of the data (images) calls for efficient representation techniques. Moreover, the knowledge required for the understanding of facial expressions is inexact in nature, mainly due to differences in various individual's idiosyncracies in style. Furthermore, this knowledge is qualitative in nature with little suitability for computer processing. For instance, the expression of joy may best be expressed as one in which the lips are stretched and lip-corners are raised. Making use of this inexact knowledge intensifies the challenge of the problem.

This paper makes the following contributions to the research in fuzzy case-based reasoning (FCBR) for intelligent facial expression recognition:

- (1) The work presents a scalable approach to recognize various facial expressions.
- (2) It proposes computationally light features to be used in the image processing module for expression recognition.
- (3) It proposes a fuzzy approach to accommodate the linguistic (ambiguous) and qualitative knowledge involved in facial expression descriptions.

- (4) It models and implements three separate classifiers: a standalone CBR classifier, a fuzzy classifier, and a hybrid fuzzy case-based classifier.
- (5) It explains testing results of the developed classifier systems using a standard facial expression dataset. The comparison of the developed classifiers is presented using receiver operating characteristics (ROC) analysis.

The rest of this paper is organized as follows: Section 2 is arranged in three subsections. In the first two subsections we give an introductory overview of CBR and FL, along with their individual merits and limitations. In the third subsection we analyze the trend towards convergence of CBR and FL in various fields. We also look at the applications using hybrid FL/CBR systems in the area of digital image processing. In Section 3 we briefly survey the basic issues and contemporary studies in facial expression recognition. This section discusses the most prominent contributions in this area. In Section 4, we present detailed architecture of the three classifier systems we have developed. Section 5 presents the details of the facial expression dataset which we have used in this study. Section 5.2 presents the detailed discussion and comparative study of the performance of the designed classifiers. We finally conclude the paper with an outlook to our future work in Section 6.

## 2. Background

### 2.1. Case-based reasoning

A CBR system emulates the natural human instinct of ‘reasoning from past experiences’ [1]. A CBR system model is built around a knowledge base of past cases, called the *case base*. Each entry in the case base contains an  $n$ -tuple of input variable values  $(u_{1i}, \dots, u_{ni})$ ,  $u_{ji}$  being defined over the space  $U_j$ ,  $j = 1, \dots, n$ , along with an  $m$ -tuple  $(v_{1i}, \dots, v_{mi})$  value of corresponding output variables with  $v_{ji}$  being defined over the output space  $V_j$ .

Given a new case  $c = (u_1^*, \dots, u_n^*)$  the case-based reasoner compares  $c$  against all cases in the case base, generating a collection of matching scores  $[m_{1i}, \dots, m_{ni}]$  for each entry in the case base, indicating the degree to which each attribute in the  $i$ th case entry matches the corresponding input attribute. The matching values in each case are aggregated to find the overall similarity of the case with the input. The case(s) with the highest level of matching is (are) used to generate the system output.

CBR has been historically represented by a four step cycle proposed by Aamodt and Plaza in 1994 [1]. The four steps, called the four REs as shown in Fig. 1 are widely acknowledged.

CBR offers several benefits for intelligent classification systems. Kolodner et al. in [29] gives a comprehensive list of CBR advantages. Some of these are summarized below:

- It facilitates the knowledge acquisition process by avoiding the time required to elicit the solutions from the experts.
- It provides a means for evaluating solutions when no algorithmic means of evaluation is available.
- CBR-based classification does not need complete understanding of the domain.
- It allows the system to learn from past experiences.

However, there are also some sensitive issues facing CBR. Some of these issues are:

- How to best represent the cases?
- How to avoid the exponential growth in size of the case base as the number of known cases increases?
- How to represent a domain containing multimedia objects like sound and images?

### 2.2. Fuzzy rule-based systems (FRBS)

An FL-based system model is a knowledge-based system comprising of rules of the form:

$$Ru_i : \text{IF } X_1 \text{ IS } F_{1i} \text{ AND } \dots \text{ AND } X_n \text{ IS } F_{ni} \text{ THEN } Y \text{ IS } G_i$$

where  $X_j$ ,  $j = 1, \dots, n$  are called the *antecedent variables*, each defined over a space  $U_j$ . Similarly,  $Y$  is the *consequent variable* defined over the space  $V$ . Each  $F_{ji}$  is a linguistic term expressed by defining fuzzy subset over the corresponding  $U_j$ . For any  $u_j \in U_j$ , the degree of membership  $\mu_{F_{ji}}(u_j)$  shows the degree to which  $u_j$  is compatible with the term  $F_{ji}$ . Similarly  $G_i$  is a linguistic term expressed by means of a fuzzy subset over  $V$ . For any  $v \in V$ , the degree of membership  $\mu_{G_i}(v)$  is the degree to which  $v$  conforms to the concept  $G_i$ .

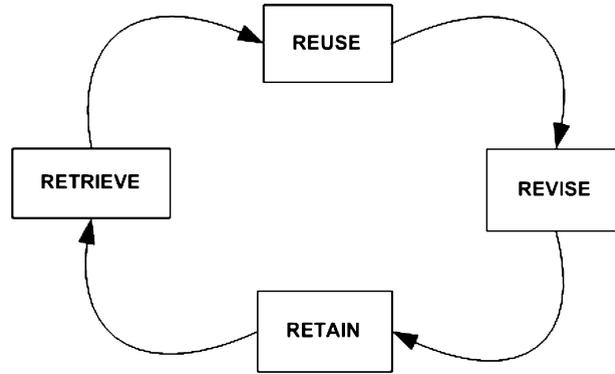


Fig. 1. CBR cycle.

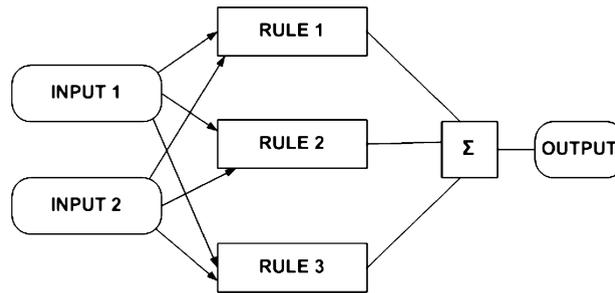


Fig. 2. Fuzzy rule-based system.

The process of reasoning with FL is as follows: given an input  $X_j = u_j^*$  we calculate the degree of compatibility of  $X_j$  with each rule (or the firing level of each rule) as:

$$\phi_i = \min \mu_{A_{ji}}(u_j^*)$$

The firing levels of all the rules are combined to calculate the system output, given by the fuzzy subset  $O$  defined over the output space:

$$\mu_O(v) = \max[\phi_i]$$

Optionally, the resulting fuzzy set is defuzzified to get a single value for the output. Fig. 2 shows the process involved in FL-based reasoning.

As a decision making technique, FL has several advantages, like:

- FL avoids the need for rigorous mathematical modeling.
- It mimics human decision making while handling vague concepts.
- FL can be used to infer from imprecise information.
- FL offers improved knowledge representation in terms of linguistic and qualitative variables.
- FL can be used to model complex, non-linear systems.

FL however has certain limitations as well. Some of these are:

- FL needs a knowledge elicitation step to gather knowledge from domain experts.
- FL does not provide a mechanism to learn either at the design phase or during use.

### 2.3. Convergence of CBR and FL

It is now a well-received argument that a combination of CBR and FL can result in systems that are more efficient and more manageable than the standalone techniques. FL can be used to build CBR systems with a tolerance for imprecision, uncertainty, approximate reasoning, and partial truth, in order to achieve tractability, robustness, low solution cost, and closer resemblance to human decision making [6].

The use of combined FL/CBR systems goes back to the early 1990s, when CBR systems with fuzzy attributes using fuzzy pattern matching were first introduced. One of the earliest hybrid CBR/FL systems is the *ARC* system [41] which uses fuzzy features to represent a prototype class of cases. The system uses a fuzzy pattern matching algorithm to find the most similar class to the input case. The *BOLERO* system [32] integrates case-based and rule-based knowledge representation for medical diagnosis. The system stores past knowledge of solved instance using linguistic terms represented by fuzzy sets. The *CARS* [3] system represents cases and problems by means of fuzzy attributes. For the retrieval step, this system calculates fuzzy similarity measure between attributes based on fuzzy algebra. The similarity is expressed by means of linguistic fuzzy terms *no-match*, *partial-match*, or *complete-match*. *PROFIT* [4] is a fuzzy CBR system for estimating value of residential property for real estate transactions. It uses fuzzy predicates to express similarity between the comparable properties. The resulting property value estimate is qualified by a fuzzy confidence measure. Further detail of FL/CBR convergence can be found in [6,49].

Image processing is a challenging field. The unique nature of data (images) calls for efficient methods of data representation. Intelligent classification systems in image processing must devise efficient methods for image interpretation to extract symbolic information from images. Existing statistical and knowledge-based techniques lack the efficiency and flexibility required for real-world applications. Some effort has been made to introduce intelligent classification methods in image processing. However, in most of these approaches, images are not processed, and the symbolic terms are user-specified [40]. An example of a scheme using intelligent methods for image interpretation is presented by the authors in [33] that uses CBR for retrieval of radiological images. The system in [18] uses CBR for detection of coronary heart diseases. The system in [17] uses CBR for interpreting abdominal CT images. In [20] CBR is used for classification of ultrasonic B-scans. The authors in [13] proposed a new way of handling formerly known image processing problems by using CBR concepts of *experience* and *adaptation*. In [34] CBR is used for scene recognition. The authors in [37] and [38] use CBR for geo-spatial image interpretation. In our previous work we proposed to use CBR for facial expression recognition [52].

Fuzzy inference has also been used in a number of applications in image processing. For example, Dave in [7] discusses boundary detection using fuzzy sets. [22] presents the use of fuzzy integrals for pattern recognition. Authors in [19] use fuzzy sets for object recognition and human motion segmentation. In our previous work, we recognized six basic facial expressions using FL [36]. Further in [24], we used FL to enhance the facial expressions in *performance-based animation*.

In the field of image processing there is a shortage of literature showing hybrid FL/CBR systems. An example is [26] where the authors combine fuzzy measures with CBR for analysis of magnetic resonance angiography (MRA) images. In our previous work [23], we used CBR with fuzzy similarity measures to recognize facial expressions.

## 3. Facial expression recognition: basic issues and contemporary approaches

Any scheme for facial expression recognition must settle on three basic choices:

- (1) The expressions to recognize.
- (2) The features to use for recognition of the selected expressions.
- (3) The classifier system to use for expression recognition.

### 3.1. Issue 1: deciding the facial expressions to be recognized

Most studies on facial expression analysis perform an emotion-based classification [39]. These systems can be classified into two main groups depending on the model of facial expressions used; systems based on the *dimensional approach* and systems based on the *category-based approach* towards emotional expressions. In the dimensional

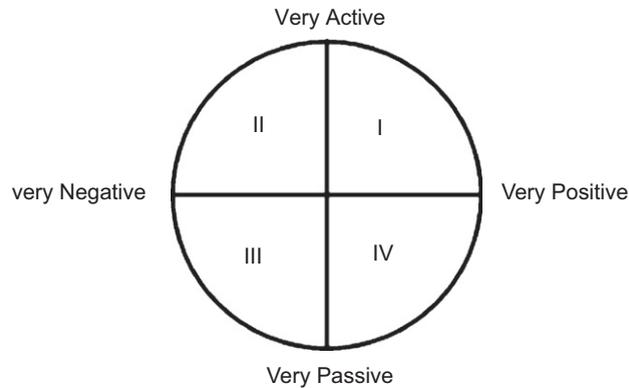


Fig. 3. The 2-D activation-evaluation space.



Fig. 4. Ekman's basic expressions.

approach, each emotion is represented as a point in an  $N$ -dimensional space. An example is the 2-D activation-evaluation space shown in Fig. 3.

The category-based approach is based on the cross cultural study of emotional facial expressions conducted by Ekman [9]. On the basis of this study, Ekman proposed six basic facial expressions, namely *anger*, *disgust*, *sad*, *joy*, *surprise*, and *fear* that are universal across all human cultures. He showed that different people exhibit similar basic facial features while producing any of these basic expressions [10]. The six basic expressions discovered by Ekman are shown in Fig. 4.

### 3.2. Issue 2: deciding the features to be used for expression recognition

The level of detail in facial features is a crucial decision for any facial expression classifier system. *Detailed* facial features make the task of classifier system easier but they require tedious image processing algorithms. These algorithms not only make system complex but also, in most of the cases, are unable to provide a universal solution because of facial diversity in different races. The increased complexity of the system requires increased computational power. This is not desirable in remote HCI interfaces. The solution to this problem is the system where feature extraction module only *estimates* the state of facial features (but with good accuracy), and intelligent classifiers are designed to cater for the inherent uncertainty in estimated facial features.

A popular set of features for facial expression recognition is the one introduced by Ekman in his facial action coding system (FACS) [11]. In this work, Ekman proposed 46 facial action units (FAUs), such that each FAU is associated with the movement of one or more facial muscles. FACS further uses A–E intensity levels for describing intensity of each FAU. An A intensity score means that not all the criteria are met for an FAU to be present; E, on the other extreme, means maximum criteria are met for an FAU to be present [14]. This representation requires us to identify and differentiate among 230 different facial states which is a difficult task. Bartlett et al. in [2] present a comparison of several machine learning algorithms using the FACS for detection of Ekman's six basic expressions. The algorithms include support vector machines (SVM), linear discriminant analysis (LDA) and principal component analysis (PCA). They obtained a classification accuracy of 93%.

Another popular feature set used by facial expression recognition systems is inspired by *MPEG-4* [35]. The philosophy behind this standard regards a video scene as a set of dynamic objects which are spatially and temporally independent [46]. *MPEG-4* specifies 84 feature points on a neutral face which provide reference for *facial animation parameters* (FAPs). Distances between these FAPs define *facial animation parameter units* (FAPUs). FAPUs allow the interpretation of FAPs in a consistent way. In order to have an *MPEG-4* compliant model, all these 84 feature points have to be known [30]. Accurate detection of these 84 feature points also presents a very difficult task mainly because the current image processing hardware and software is not advanced enough to produce precise results. Examples of systems using these features include [36,24].

### 3.3. Issue 3: deciding the classification scheme

Different classification schemes such as PCA, SVMs, artificial neural nets (ANNs), fuzzy neural nets (FNNs), FL, and self organizing maps (SOM) have been used by authors in [43,2,27,28,45,25,47,21] for facial expression recognition. However, most of these approaches have used complex feature representations as explained earlier.

In the next section, we present our proposed approach in which we try to maintain an appropriate balance between the above-mentioned issues.

## 4. The proposed approach

In our approach we use Ekman's six basic emotions as the facial expressions to be detected. In order to reduce the complexity of the feature extraction module, we have used a simplified feature extraction scheme using only eight basic *facial action elements* (FAEs). A region extraction scheme detects the regions of all FAEs. The extracted regions are used to estimate the state of FAEs in the range [0,1]. We call this state the *facial action value* (FAV).

Initially we built a standalone CBR system for expression classification. Later, exploiting the inherent uncertain nature of the FAEs, we used fuzzy sets for measurement of facial action. In the third stage, aiming to impart adaptability to the system we enclosed the fuzzy rule base in a case-based framework that can tune itself by learning from experience. This hybrid approach shows better experimental results as shown in Section 5.2. Details of the approach are presented below:

### 4.1. Feature extraction module

Input to the feature extraction module is a colored image from which face and features are to be extracted. The first step is to localize the region of interest (ROI), i.e. the face, on the input image. For face extraction, we detect the skin colored pixels in the image. There are several advantages of using skin color as the first step in ROI detection, e.g.

- It allows fast processing.
- It is robust to geometric variations in face pattern.

Different color spaces can be used for skin color detection, e.g. *HSI*, *RGB*, *YCbCr*, etc. We use the *normalized RGB* color space [16] for our purpose. A pixel is classified as skin pixel if [15]

$$\frac{r}{g} \geq 1.185, \quad \frac{rb}{(r+b+g)^2} \geq 0.107 \quad \text{and} \quad \frac{rg}{(r+b+g)^2} \geq 0.112$$

where  $r$ ,  $g$ , and  $b$  are, respectively, the normalized red, green, and blue components of the image. Connected components are grown from the skin colored pixels using *floodfill algorithm*. The floodfill algorithm is based on the concept of pixel neighborhood (Table 1). A pixel in the neighborhood of pixel  $p$  that possesses the same attribute (i.e. being a skin color pixel) is considered to belong to the same region as  $p$ .

The region growing process produces a black and white image, with white regions indicating the detected skin regions, and the black regions indicating the background. Any holes in these regions are filled, and face candidates are selected from these connected components using geometric properties of the human face. Specifically, two rules are applied at this stage. First, the region should not be smaller than a threshold, and second, the height to width ratio should be near the golden ratio (i.e. 1.62). Regions satisfying both these conditions are selected as potential face candidates. An example is shown in Fig. 5.

Table 1  
Pixel neighborhood

1	1	1
1	$p$	1
1	1	1

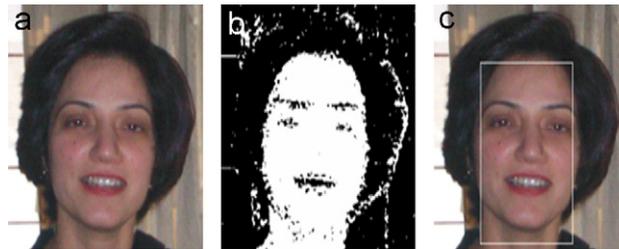


Fig. 5. (a) Input image, (b) skin pixels, and (c) detected face.

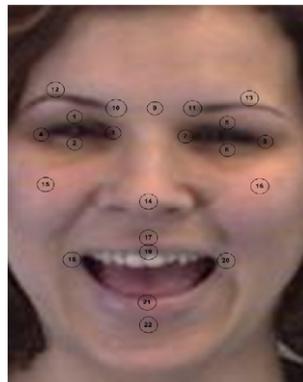


Fig. 6. Feature points for feature estimation.

Once the face is detected, FAEs are extracted from the face area. Eight basic FAEs considered for expression output are: *eyes, eyebrows, forehead, nose, teeth, cheeks, lips, and chin*. Several reference points are defined on the face, and their mutual distances are used for finding specific FAVs (see Fig. 6).

For locating the FAEs, first of all, the image is binarized using standard iterative thresholding. Morphological image processing is performed on the thresholded image to enhance the feature regions. The resulting image is divided vertically into a number of slits of pre-determined sizes. Moving down from the top, the first slit with a relatively small average intensity is the eyebrow area. The next slit with a low average intensity is the eye region. The next slits possessing similar characteristics are the nostril and mouth regions, respectively.

After detecting the vertical positions of the features, horizontal boundaries of the features are calculated using standard vertical integral projections. We use the edge image  $E(x, y)$  of a feature slit for this purpose. The vertical integral projection of a slit is calculated as

$$V(x) = \sum_{y=1}^N E(x, y)$$

where  $x$  is the column index,  $y$  is the row index, and  $N$  is the height of the slit under consideration.

By observing local maxima in the vertical integral projection of the slits, the horizontal positions of the facial features are found. An example is shown in Fig. 7 where the local maxima are showing the eye corners.

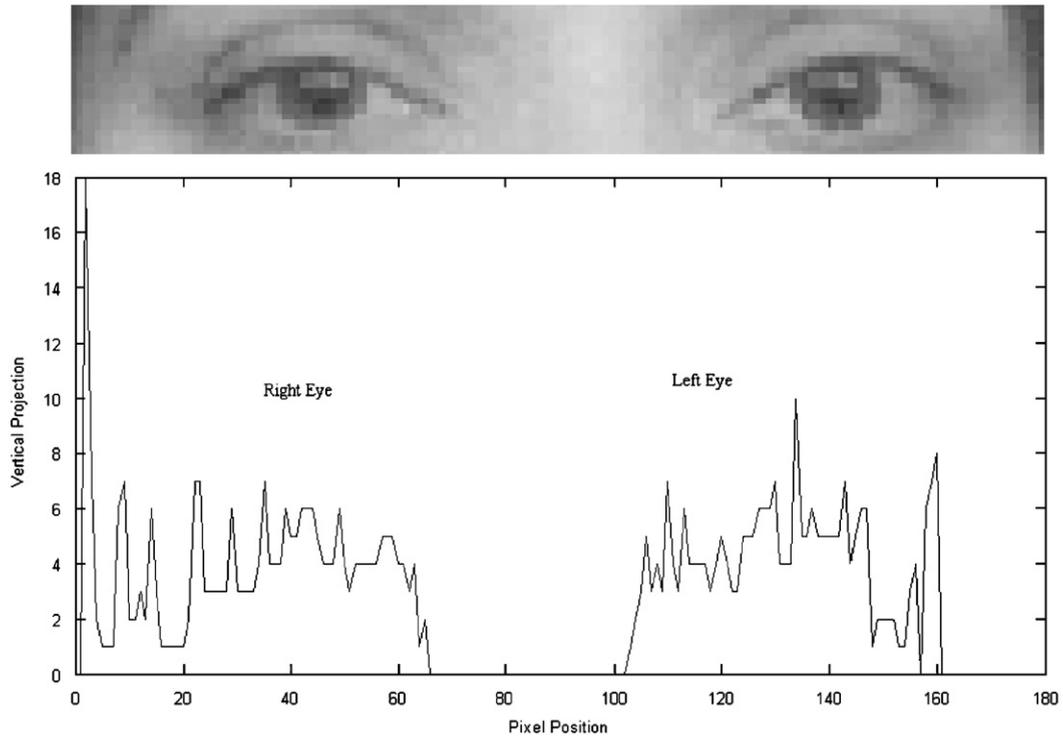


Fig. 7. Feature position detection using vertical projection.

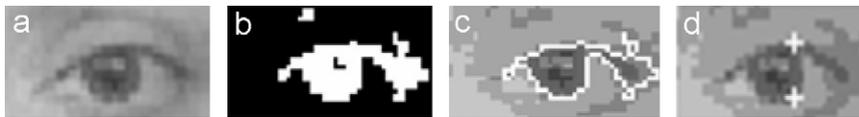


Fig. 8. (a) Eye image, (b) threshold eye image, (c) contour traced eye image, and (d) detected points.

The above process locates the lip corners as well as eyebrow endpoints and the eye corners. The top and bottom control points of the lips are found by locating two bright pixels roughly between the left and right lip corners in the top and bottom parts of the thresholded lip image. In order to locate upper and lower control points of the eyes, we find the contour of the upper and lower eyelid region to obtain the set of points  $(x_{1u}, y_{1u}), \dots, (x_{n_u}, y_{n_u})$  for the upper lid and  $(x_{1l}, y_{1l}), \dots, (x_{n_l}, y_{n_l})$  for the lower lid. Euclidean distance is calculated for each pair of corresponding points on the upper and lower lid  $(x_{i_u}, y_{i_u})(x_{i_l}, y_{i_l})$ . The two points giving the maximum distance measure are taken as the lower and upper midpoints of the eye provided they lie near the midpoint of the two already calculated eye corners. This process is shown in Fig. 8.

The above extracted feature points are used to FAVs. Table 2 lists all the parameters for FAE state calculation, where  $s(a, b)$  denotes the Euclidean distance between points  $a$  and  $b$  marked in Fig. 6.

#### 4.2. Classifier systems

All three classifiers presented below utilize the facial feature extraction module described in the above section. The input facial features are in the form of a row vector, each element of the vector representing a single FAV. The specific details of each individual classifier are as follows:

Table 2  
FAE state extraction parameters

S. no.	FAE	FAV	FAV parameters
1	Eyes	$V_1$	$s(1, 2)$ and $s(5, 6)$
2	Eyebrows	$V_2$	$s(10, 11)$ , $s(1, 12)$ , and $s(5, 13)$
3	Forehead	$V_3$	Gradient calculation above the line (10,11)
4	Nose	$V_4$	$s(9, 14)$
5	Lips	$V_5$	$s(17, 22)$ and $s(18, 20)$
6	Teeth	$V_6$	Brightness distribution along the line (17,22)
7	Cheeks	$V_7$	$s(15, 4)$ , $s(15, 2)$ , $s(15, 3)$ , $s(16, 8)$ , $s(16, 6)$ , $s(16, 7)$
8	Chin	$V_8$	Gradient below the point 22

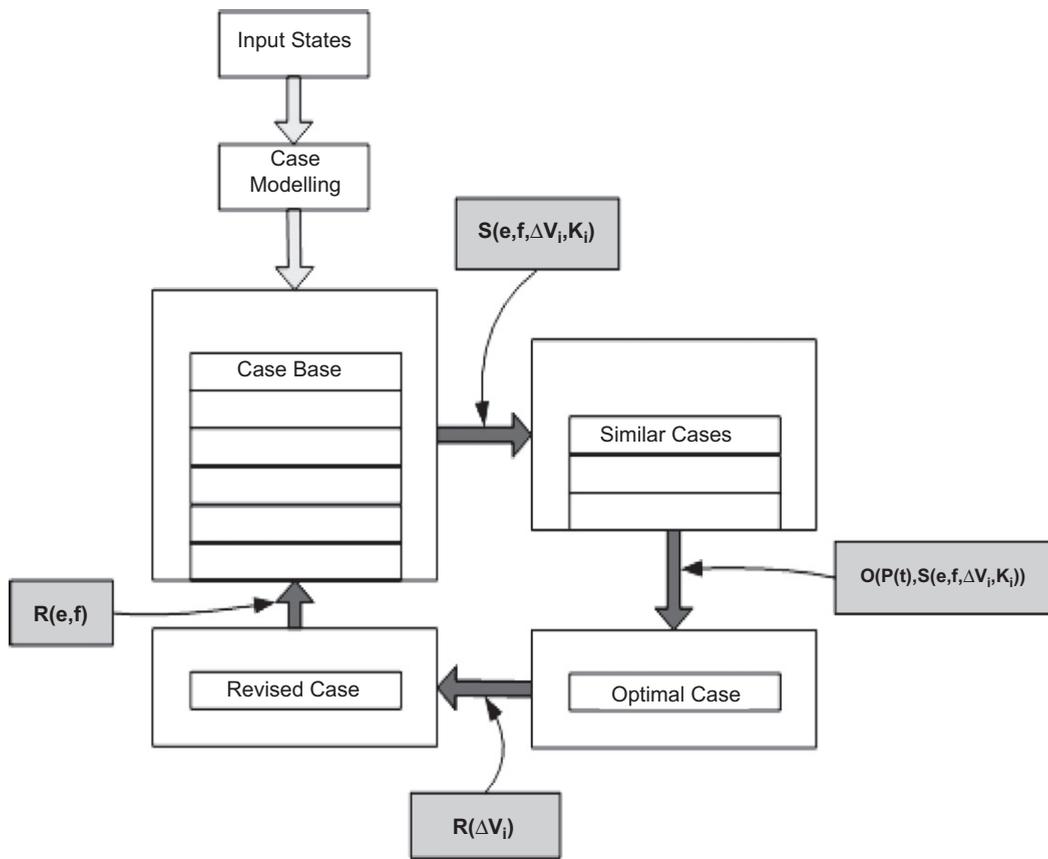


Fig. 9. CBR cycle for facial expression recognition.

4.2.1. Classifier based on CBR

The architectural details of the CBR-based classifier system for facial expressions are shown in Fig. 9. The individual elements of the system are described next:

4.2.1.1. Case definition The important constituent of a case are FAVs for all the FAEs. A case also has an associated efficiency index ( $e$ ) which indicates its success rate in the past. A case entry also contains the frequency of past use ( $f$ ), which is also an important parameter. The output of a case is an expression  $E$  from Ekman’s categories. We assign an integer identifier to each basic expression as shown in Table 3. Case structure is shown in Table 4.

Table 3  
Integer labels assigned to basic expressions

Expression	Integer label
Anger	0
Disgust	1
Sadness	2
Neutral	3
Joy	4
Surprise	5
Fear	6

Table 4  
Case structure

Parameter	Symbol	Remarks
Efficiency index	$e_i$	Initially set to zero. Increases linearly
Frequency	$f_i$	No. of times this case has been activated
Eyes	$V_{1i}$	FAV for eyes
Eyebrows	$V_{2i}$	FAV for eyebrows
Forehead	$V_{3i}$	FAV for forehead
Cheeks	$V_{4i}$	FAV for nose
Nose	$V_{5i}$	FAV for teeth
Lips	$V_{6i}$	FAV for cheeks
Teeth	$V_{7i}$	FAV for lips
Chin	$V_{8i}$	FAV for chin
Expression output	$E_i$	Detected expression; ranges from 0–6

4.2.1.2. *Case formation* The preceding facial image processing and feature extraction module assigns each FAE a value (i.e. the FAV). Every input situation is modeled as a case for comparisons with past cases in the case base.

4.2.1.3. *Similarity assessment* For calculation of similarity between two cases we define a *distance measure D* which is a function of the following three factors:

- Efficiency index  $e_i$ .
- Frequency of occurrence  $f_i$ .
- Difference of FAVs in the input case and the considered case from case base.

$D$  is calculated as below:

$$D(U_i, U^*) = \frac{\alpha \sum_{j=1}^8 \left| \beta K_j \left[ \frac{\Delta V_{ij}}{V_{ij}} \right] \right|}{e_i f_i \sum_{j=1}^8 K_j}$$

where  $\alpha$  and  $\beta$  are normalizing constants,  $K_1, \dots, K_8$  are controlling weights which decide the relative importance of the eight FAs. They are specific for every person’s profile. Every person has some features that are of characteristic nature, i.e. their contribution in the final expression output is relatively greater [53].  $\Delta V_i$  is the difference between the observed and the stored FAV in the case.  $D(U_i, U^*)$  returns an integer as output whose value is inversely proportional to similarity. Hence, all the cases with the distance values lesser than a threshold set will be marked as similar cases.

4.2.1.4. *Optimized distance-based case selection* Optimization is performed to get an optimal measure of distance of the input case to a selected case in the case base. Our proposed optimization function analyzes two important factors: *present context* and *distance between the two cases*.

By the term *present context*, we mean the expressions detected on the previous runs of the system on the same video sequence (called the *states*). It is an important factor when selecting the optimized case from the similar cases. Present context can be extended back to any number of previous states. But for simplicity, we use three previous states for

defining the present context. We also define a *novelty function*  $N(t)$  where  $t = 0, 1, \dots$  is the current state number. The output of  $N(t)$  is an integer whose value approach zero if the previous three states have the same expression output as the proposed solution in the current case base entry.

Distance optimization function is given by the following formula:

$$O(U_i) = \gamma N(t) + \eta D(U_i, U^*)$$

where  $\gamma$  and  $\eta$  are the constants defining the relative weight of  $N(t)$  and  $D(U_i, U^*)$ . Their values lie in the range  $[0,1]$ . Hence the output value of  $O(U_i)$  approaches zero in case of an ideal match.

**4.2.1.5. Revision of result** The revision module revises the result in the current case entry depending upon the relative difference in values of FAEs, i.e. the FAVs. This revision is necessary because an exact match between values of action elements is quite rare. The revised expression output is determined as below:

$$E_{i\_revised} = E_i + \zeta \left[ \frac{\sum_{j=1}^8 |K_j [\Delta V_{ij} / V_{ij}]|}{\sum_{j=1}^8 K_j} \right]$$

where  $E_i$  is the expression output of current case,  $E_{i\_revised}$  is the revised expression output, and  $\zeta$  is the normalizing constant.

**4.2.1.6. Learning function** A case from the case base is updated after every usage. Efficiency index ( $e_i$ ) and frequency of occurrence ( $f_i$ ) of the case are updated by a learning function that executes the following two tasks:

$$f_{i\_updated} = f_{i\_old} + 1$$

$$e_{i\_updated} = e_{i\_old} \pm \lambda / f_{i\_updated}$$

where  $\lambda$  is a constant and  $\pm$  accounts for increase or decrease in efficiency index ( $e$ ) for success with respect to success or failure, respectively.

#### 4.2.2. Classifier based on fuzzy rule-based system (FRBS)

Structure of the fuzzy rule-based system (FRBS) design for facial expression recognition is shown in Fig. 10. The details of implementation are given below:

**4.2.2.1. Input membership functions** The system input comprises of the FAV values enumerated in Table 2. The FAV values are fuzzified by the input membership functions (MFs) of the form:  $\mu_{X_i}(V_i) : V_i \rightarrow [0, 1]$  where

$$X_i \in T_i$$

with  $T_i$  being the term set for the feature corresponding to  $V_i$  in Table 2. The members of  $T_i$  for various values of  $i$  are given below:

$$T_1 = \{Pressed\_Closed, Normal, Extra\_Open\}.$$

$$T_2 = \{Centered, Normal, Outward\_Stretched\}.$$

$$T_3 = \{Down\&Small, Normal, Stretched\&Bigger\}.$$

$$T_4 = \{Flat\&Stretched, Normal, Filled\&Up\}.$$

$$T_5 = \{Normal, Radical\}.$$

$$T_6 = \{Pressed\_Closed, Normal, Open\}.$$

$$T_7 = \{Not\_Visible, Slightly\_Out, Extra\_Open\}.$$

$$T_8 = \{Normal, Radical\}.$$

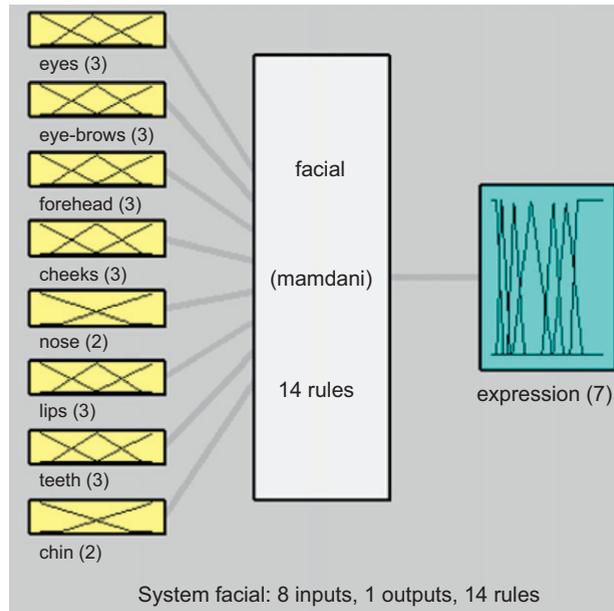


Fig. 10. FRBS architecture.

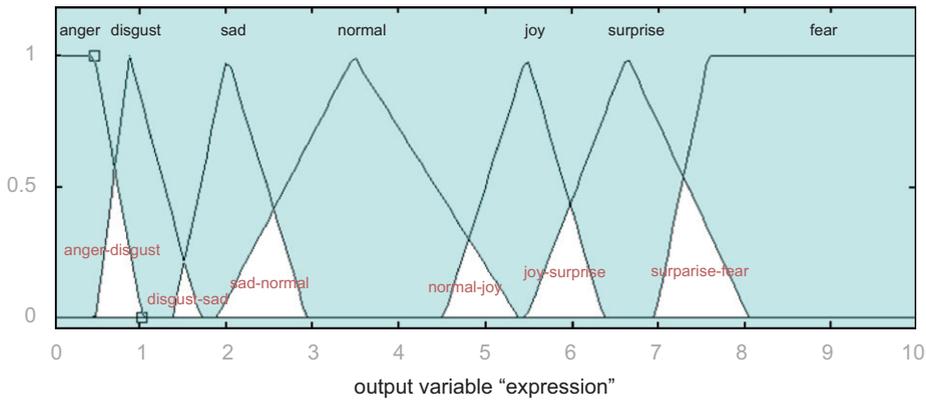


Fig. 11. MFs plot for ‘expression’ showing overlap regions [36].

4.2.2.2. *Output MFs* The system has one output linguistic variable, *expression*, having seven terms for the seven basic facial expressions. The distinctive feature of our system is the design of ‘expression’ MFs. They are defined on the scale 0–10. The grouping of the facial expression MFs is shown in Fig. 11, and justified below:

Starting from the extreme left, *anger* and *disgust* expressions are commonly confused for similarity. It is evident from survey in [50] that these two categories are overlapping. So, it makes sense to group them together. Authors in [50] also show that *sad* expression is also confused with *disgust*. However, the percentage of the surveyed people who confused *disgust* with *sad* in presented facial expressions was lesser than those who confused *anger* and *disgust*. So it makes sense that overlapping area for *disgust–sad* is lesser than that of *anger–disgust*. In the middle *normal* (or *neutral*) facial expression bridges *sad* and *joy* expression MFs. Facial features tend to become like those of *surprise* as the *joy* becomes extreme and it becomes difficult to distinguish the two expressions [8]. Studies show that *fear* is often misrecognized as *surprise* [42]. Making use of this finding, towards the extreme right in the sequence of MFs, *fear* overtakes *surprise*.

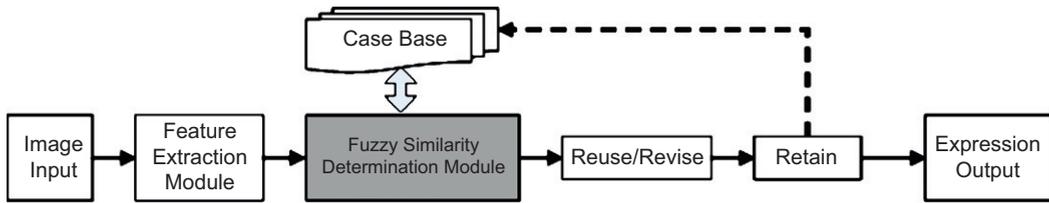


Fig. 12. Fuzzy case-based reasoning system.

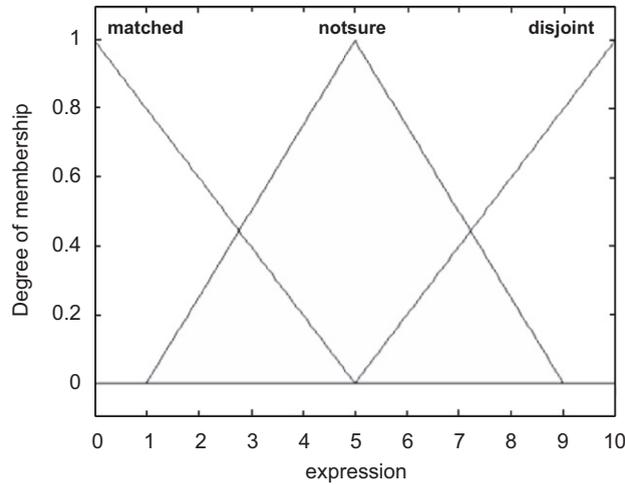


Fig. 13. Output MFs.

Output ‘expression’ membership functions are given as

$$\mu_O(\text{Expression})$$

where

$$O = \{\text{Anger, Disgust, Sad, Normal, Joy, Surprise, Fear}\}$$

#### 4.2.3. Classifier based on FCBR

The hybrid FCBR system can be viewed as a CBR system with FRBSs as the cases in case base [23]. In this way, FL is embedded into our CBR system for improved case retrieval. Architecture of FCBR system is shown in Fig. 12.

Similar to the standalone CBR and FRBS classifiers, the input to the combined FCBR system comprises of the FAVs from the feature extraction module. Fuzzified FAVs are fed to all the FRBSs in the case base for similarity determination. FRBSs in each case entry rate the distance of an input case to a particular case (in the case base) with the help of the output linguistic variable called *distance*. This variable contains three fuzzy terms in its term set, altogether spanning the range 0–10. Thus, the output can be represented by the following MF:

$$\mu_O(\text{Distance})$$

where

$$O = \{\text{matched, notsure, disjoint}\}$$

The structure of the MFs for *distance* is shown in Fig. 13. If distance approaches 0, then this means the retrieved case entry is similar to the input case and the solution provided in the case entry may be used to solve the problem.

The efficiency index (*e*) and frequency of use (*f*) as explained in the section on CBR-based classifier are also considered while calculating the distance of an input case from a particular case entry. Thresholding is done to select

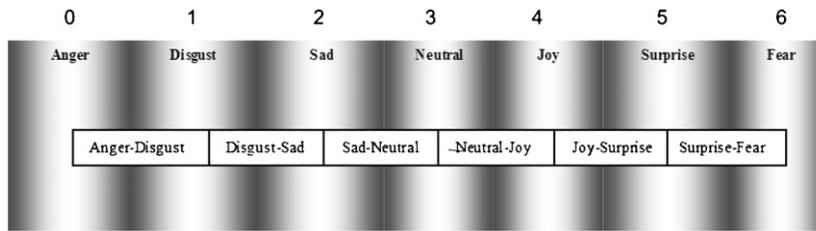


Fig. 14. Model for overlapping expressions.

the most similar (i.e. the least distant) cases, which are then passed on to the optimization function. The optimization function selects the optimized function on the basis of the similarity as well as the present context. The revision and learning functions are the same as used in standalone CBR system.

The defuzzification scheme used by the FRBSs is *mean of maxima (MoM)*. This particular method was chosen to make explicit decision about the similarity of input case with a particular case entry. The FRBS indicating the closest similarity is used to produce the system output.

The above procedure is sufficient in determining a single expression that matches the input image. However, we observe that in real life facial expressions rarely occur alone. Most of the times, facial expressions occur as blends of different basic expressions rather than a single basic expression. We have enhanced our FCBR system to recognize blends of basic expressions in addition to single expression. For this purpose we use a linear space as shown in Fig. 14.

This space is inspired by output MF model of standalone FRBS system (see Section 4.2.2). Based on this framework, we propose a new algorithm to identify overlapping expressions. This algorithm takes the single expression output by the case entry having an FRBS output indicating the closest match to the input image. The algorithm then uses the model in Fig. 14 to detect a single other expression (on the left or on the right of the detected expression) that is blended with the just detected single expression in order to produce the expression depicted in the input image. The algorithm is presented below:

---

#### Algorithm for overlapping expressions.

---

```

i = Expression Output % The single expression output of the FCBR based on the closest similarity
OverlapExpression=3 % Initially assume the blend expression is Neutral
x = FRBSEvaluate(i + 1) % Check similarity of the input with the expression to right of the detected expression as shown in the
    % linear space. Store result in x
y = FRBSEvaluate(i - 1) % Check similarity of the input with the expression to left of the detected expression as shown in the
    % linear space. Store result in y
if (x < y) % If expression on the right has closer similarity
OverlapExpression = x % Expression on the right is the blend expression
else if (x > y) % If expression on the left has closer similarity
OverlapExpression = y % Expression on the left is the blend expression
return OverlapExpression % Return the blend expression

```

---

## 5. Evaluation

### 5.1. Dataset

The system was evaluated using the FG-NeT database from the Technical University Munich [48]. The database has been generated as part of the European Union project *FGNeT* (face and gesture recognition research network). This is an image database containing face images showing a number of subjects performing the six different basic emotions defined by Ekman and Friesen. The covered emotions include *joy*, *disgust*, *anger*, *fear*, *sadness*, *surprise*, and *neutral*. The database has been developed in an attempt to assist researchers who investigate the effects of different facial expressions. It contains images gathered from 18 different individuals. Every individual has performed six basic expressions (besides the *neutral*) three times. All together this gives an amount of 378 video sequences. The video was

	P	N
Y	True Positives	False Positives
N	False Negatives	True Negatives

Fig. 15. The 2 × 2 ROC confusion matrix (redrawn from [12]).



Fig. 16. *tp rate*, *fp rate*, and *accuracy* plots for CBR system.

captured while simultaneously displaying different emotion-eliciting scenes to the subjects. Depending on the kind of emotion, a single recorded sequence can take up to several seconds. The images were acquired using a Sony XC-999P camera equipped with an 8mm COSMICAR 1:1.4 television lens. A BTTV 878 frame grabber card was used to grab the images with a size of 640 × 480 pixels, a color depth of 24 bits and a frame rate of 25 frames per second. Later on, the images were converted into 8 bits JPEG-compressed images with a size of 320 × 240.

## 5.2. Methodology

### 5.2.1. Performance metrics

Fig. 15 shows some of the metrics used for performance evaluation of the developed classifiers. For most purposes, we have used a 2 × 2 confusion matrix as is used for binary classifiers. This matrix defines the ROC parameters used for classifier evaluation.

With reference to the figure, the relevant metrics include the false positive rate (*fp rate*), the true positive rate (*tp rate*), and the *accuracy*. These metrics are defined below:

$$tp\ rate = \frac{TP}{P}, \quad fp\ rate = \frac{FP}{N}, \quad accuracy = \frac{TP + TN}{P + N}$$

### 5.2.2. Results

We tested our designed systems (CBR, FRBS, FCBR) using the FG-NeT database, and calculated the ROC parameters for all.

The ROC parameters for CBR were calculated considering the problem of the basic expression output as the binary classification problem. For example, if a *happy* expression is the input and *happy* is the expression output, then this is considered true-positive for *happy* and true-negative for rest. Similarly, if a *happy* expression is the input and (say) *surprise* is the expression output, then this is considered false-negative for *happy*, false-positive for *surprise* and true-negative for rest. The *tp rate*, *fp rate*, and *accuracy* plots for the CBR-based classifier are shown in Fig. 16.

A plot of the *tp rate* vs the *fp rate* for the classifier is shown in Fig. 17.

The *tp rate*, *fp rate*, and *accuracy* parameters for FRBS were also calculated using the same principle as applied for CBR system. The plots of these parameters are shown in Fig. 18.

A plot of the *tp rate* vs the *fp rate* for the classifier is shown in Fig. 19.

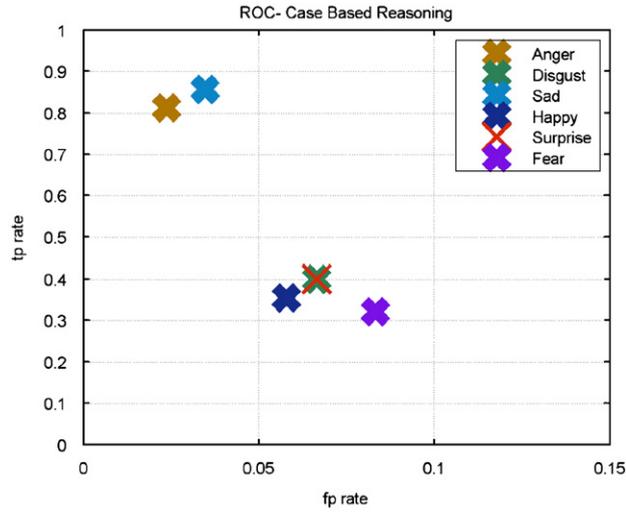


Fig. 17. ROC plot for CBR system.

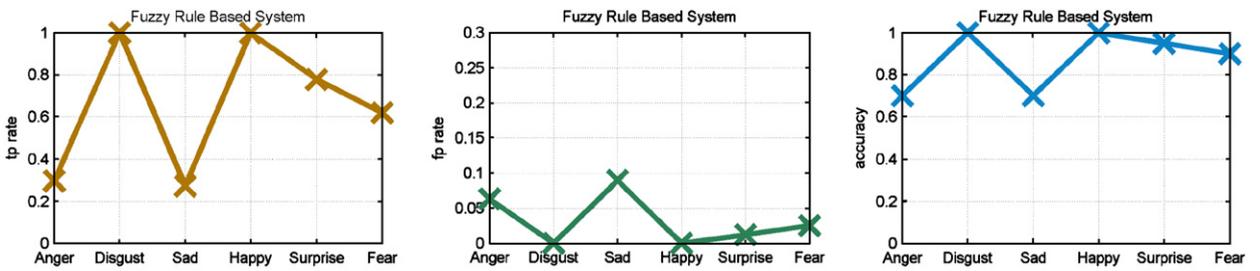


Fig. 18. *tp rate*, *fp rate*, and *accuracy* plots for fuzzy rule-based system.

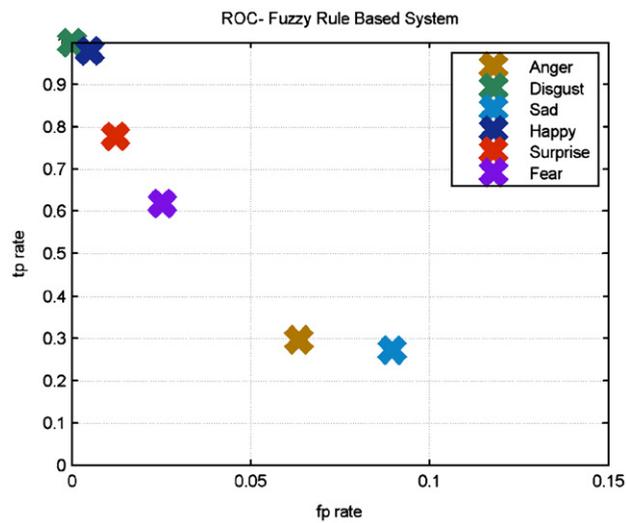


Fig. 19. ROC plot for FRBS.

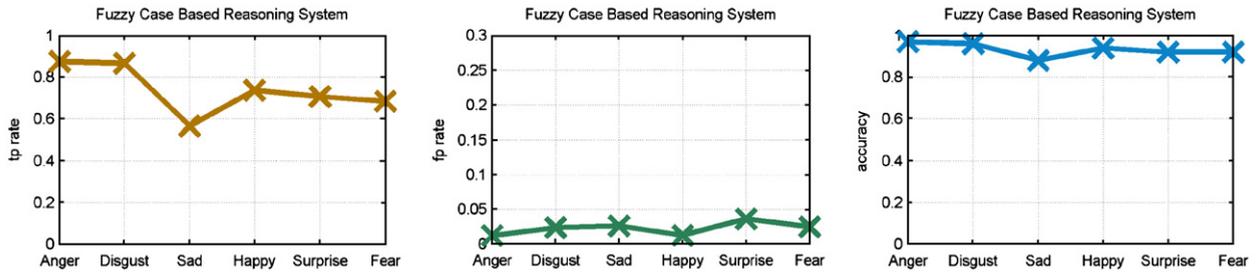


Fig. 20. *tp rate*, *fp rate*, and *accuracy* plots for FCBR system.

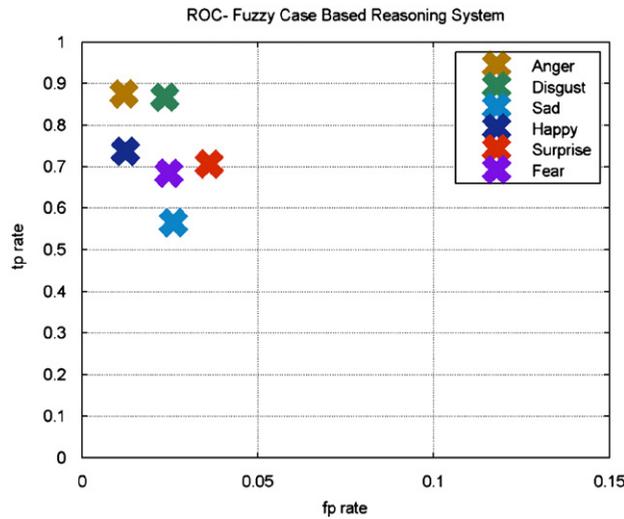


Fig. 21. ROC plot for FCBR system.

The *tp rate*, *fp rate*, and *accuracy* plots for the FCBR-based classifier are shown in Fig. 20. A plot of the *tp rate* vs the *fp rate* for the classifier is shown in Fig. 21.

### 5.3. Discussion

It is clear that the ROC points for all basic expressions cluster near the ‘perfect classification point’ (0,1). A comparison of these results to the standalone CBR and FRBS systems clearly shows that FCBR is able to combine the useful properties of both methods to produce more accurate results. For instance, in case of the *tp rate* and the *accuracy* one can notice that while the standalone systems are able to show good performance at isolated instances, the FCBR system shows consistently good performance for all expressions, never giving the worst performance of the three approaches. Although the FRBS system gives the best performance in case of four out of six basic expressions (i.e. for *disgust*, *happy*, *surprise*, and *fear*), still it faces a lot of problem in understanding the remaining two expressions (*sadness* and *anger*), giving the worst performance of the three classifiers. On the other hand, the *accuracy* of the FCBR system never falls below 85% for any expression.

We have also compared the performance of our system with other approaches presented in literature. The focus of our system is on reducing design complexity and increasing the *accuracy* of expression recognition. While the FACS-based approaches need 46 FAUs and MPEG-4-based approaches need to keep track of 84 feature points, the scheme proposed in the current work needs only eight FAEs. FNN have been used by [25] for ‘personalized’ recognition of facial expressions. The success rate achieved by them reaches 94.3%, but only after the training phase which clearly is a tedious job. Bartlett et al. in [2] present a comparison of several machine learning algorithms using the FACS for detection of Ekman’s six basic expressions. The algorithms include SVMs, LDA, and PCA. They obtained a

Table 5  
Comparison of the proposed hybrid system with other systems in literature

Expression	DP (%)	HMM (%)	CBR (%)	FRBS (%)	FCBR (%)
Surprise	91.66	66.66	80	95	100
Disgust	4.16	41.67	80	100	82
Happy	91.66	96.66	76	100	95
Anger	87.50	36.11	95	70	100
Fear	79.16	63.89	75	90	80
Sad	8.33	27.78	95	70	85

classification accuracy of 93%. Cohen [54] used hidden Markov models (HMM) as well as dynamic programming (DP) for facial expression recognition. Katoh and Fukui [21] used SOM for this purpose. Other techniques like HMM are further used in [51]. The comparison results are shown in Table 5.

## 6. Conclusion

In this paper we have used fuzzy case-based reasoning (FCBR) for facial expression recognition. Facial features are extracted from a static image. These features are fed to the classifier system for recognition of facial expression. We first developed standalone CBR classifier and FRBS classifier. Both of the classifiers have their own advantages and disadvantages. To combine the advantages from both domains we then developed an FCBR system. FCBR system has shown its potential in the integration of CBR with FRBS for improved case retrieval. This integration is especially useful in classification problems where it is hard to find crisp distinction between two classes. Facial expression recognition is a very good example of such problem domains where the use of fuzzy logic along with CBR not only improves the system performance but also the linguistic variables used in fuzzy rules provide an additional insight into the working of system.

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